FEEG1003 : Thermofluids

An Introduction to Thermodynamics and Fluid Mechanics for Year 1 Engineering Students

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Lecture 1: What is the subject of "Thermofluids"?

"Thermo" + "Fluids"

- Thermodynamics : the state of a *fluid* and its use to convert energy.
- Fluid mechanics : the static and dynamic behavior of *fluids*.

A *fluid* is a generic term encompassing liquids and gases.

Ways in which we *use* fluids include...

Transport : Things (Aircraft, ships, cars, people) move through a fluid...
Handling : Fluids moving through things (pipes, channels, valleys...)
Energy Conversion : Engines, Fridges, Dams, Wind Turbines...
Environment : Pollution emission and dispersion, climate change..

We solve one or more of the following equations..

Mass : Mass conservation equation...
Momentum : Newton’s 2nd Law
Energy : First Law of Thermodynamics.

Course Outline by Section.

Section 1 : Tools of the Trade

Revision and expansion of the knowledge we need to build fluid machinery of various sorts. Groundwork !!

Section 2: Fixed Mass Analysis

Conservation laws for a fixed mass of fluid that has no viscosity (friction) : Energy conservation (transfer) only. Study how to convert thermal energy to motive work (engines) using gases (compressible fluids).

Section 3: Inviscid Fixed Volume Analysis

With flow, and again no viscosity. Thus mass, momentum and energy conservation are important and now defined for a fixed volume. Motion is determined purely by applying a force (Newtons 2nd Law).

Section 4: Viscous Fixed Volume Analysis

Now consider processes where both viscous and inertial forces are important, with incompressible fluids. Viscous forces are usually important near walls.

What is FEEG1003 going to teach you to do...

Aim

Provide an understanding of fundamental aspects of the physics of fluid flow and energy transfer and to develop tools to analyse simple engineering fluids systems.

Having successfully completed the module..

- The properties of thermofluid, methods of analysis, including conservation principles for mass, momentum and energy.
- To analyse a thermofluids problem, decide what the form of the governing equations are.
- Translate a paragraph of text describing a problem into a set of mathematical equations.
- Solve the problem !

To understand the question, define the question mathematically, and then to solve it. (More detail on this in the *Module Specification* document on the Module Blackboard site.)

Learning Methods and Materials.

Staff time includes several levels of granularity..

- Lectures and videos (~ 2 per week), to 100s.
- Structured Tutorial Sessions (~ 1 per week), to 10s.
- One to one help (1 hour per week, per tutorial group), to 1s.

Your learning activities include..

- Directed reading (of lecture material), textbook sections. Video Resources.
- Problem solving (workbook questions, tutorial classes).
- Practical classes (laboratories).

Course Materials

- Course *notes*, these slides.
- References to material in textbooks.
- homework questions, with answers but without solutions.
- Feedback in tutorial sessions, laboratory, recorded solutions.

Aim of Lectures, Tutorial Sessions, 1-2-1 Time

Lecture Objectives

- Provide an *introduction* and an overview of a subject/physics/phenomena.
- Explain where the governing equations come from, what they are.
- Make clear what the assumptions are in deriving them.
- Provide the toolkits you need to solve problems.

Tutorial Objectives

- Train you to set up and solve problems.
- Worked solutions will be provided every week by cohort academics.
- Learn the tricks of the trade.
- Small groups, ask lots of questions !

1-2-1 Objectives

- Your opportunity to get help if you are stuck on a problem. Individual help !
- Ask the right question, and you will get a good reply.
- one hour per week is available.

Laboratories make Engineering Come to Life..

Engineering is a hands-on subject

- This is a difficult subject. Hands on learning, running experiments is fantastic learning method.
- It is also really important that the lecture, tutorial and laboratory focus is syncronised.
- *Hands on* means small groups is a necessity.

On a cohort of 400-500 this is not easy, and the faculty has invested heavily as follows.

- For most experiments we have purchased 12 multi-purpose "benches" onto which we can run most of the experiments.
- Set the 12 benches up for a single experiment, and get the entire cohort through, in small groups, in a few weeks.
- The experiments and the lecture content stay synchronised and each reinforces the other.

As far as I am aware, this is unique in the UK.

- There are 6 different experiments, to support a different aspect of the course.
- 1 is a demonstrations.
- The lab reports are designed to focus on the physics and avoid too much tedious report writing.

Laboratory Details

Lab	Subject
L1	Flow Visualization
L2	Ideal Gas laws and Processes
L3	Heat Engines
L4	Hydrostatic Force
L5	Conservation of Energy (Bernoulli)
L6	Conservation of Momentum (Jet Impact)

- Each lab should take 1 hour.
- Time is tight. You must prepare and read up beforehand.

NOTE: If you miss your lab slot you have no right to another. It is very important to make use of your allocated lab slots.

Your expectations, and my expectations..

A 15 Credit Module means 150 hours of your time

- Roughly 1/2 is timetabled. The other half is self-study time.
- Lectures are very intensive (25 hours) - reading up after the lecture is strongly recommended.
- The lecture slides are your notes - expand on them
- Keep trying the problems, some are easy, some are not.
- ASK QUESTIONS in your tutorial groups and especially 1-2-1 time.
- Emailed queries do not work so well.
- Course queries should be directed to me, via blackboard. The question, and my reply will always be distributed to all students on the course, via blackboard.

Text Book Recommendations

There is no primary text for this course, there are a *vast* range available. Here I list some I think are very good. And one suggestion.

Fluids Texts

- Crowe et al. ’Engineering Fluid Mechanics’, 9^{th} ed. (better than 10^{th}). SI version. Fairly introductory but great clarity. $\sim$£50 new.
- Fox, Pritchard and McDonald, "Introduction to Fluid Mechanics", 7^{th} ed, SI units. Overall very good. $\sim$£30.
- Cengel and Cimbala, "Fluid mechanics: fundamentals and applications", 2^{nd} ed. Pretty good, comes with a DVD.

Thermodynamics Texts

- Cengel YA and Boles MA, ’Thermodynamics - an engineering approach’, 7^{th} ed., SI units, McGraw Hill. - very good. $\sim$£45.
- Rogers and Mayhew also good $\sim$£65.

Fluid Mechanics + Thermodynamics

- Shrimpton JS, An Introduction to Engineering Thermofluids, 2015, ~ 180 pages, £10 locally, also on Amazon.

Academic Staff allocated to this module

Lecturers

- John Shrimpton (module lead), Luke Myers, Davide Lasagna, Ivo Peters

Structured tutorial sessions are theme specific

Aero	Davide Lasagna, Ivo Peters
Acoustics	Alan McAlpine
Civil	Luke Myers
Mech	Anatoliy Vorobev, Xunli Zhang, Dmitry Bavykin, Dario Carugo
Ship	Joe Banks

Laboratories

- Luke Myers (organiser), Blair Thornton (Timetabling) and ship science staff (on the ground)

- In addition we have ≈ 20 PhD students to help out in the labs
- All exams marked, second marked and moderated by academic staff.
- You can find our contact details on the module blackboard site.

Lectures and Lecture Dates

Semester 1	Dates (w/c)	Lecture 1	Lecture 2	Semester 2	Dates (w/c)	Lecture 1	Lecture 2
Week 1	1/10/18	(none)	(none)	Week 18	28/1/19	20	21
Week 2	8/10/18	1	2	Week 19	4/2/19	22	23
Week 3	15/10/18	3	4	Week 20	11/2/19	24	25
Week 4	22/10/18	5	6	Week 21	18/2/19	26	27
Week 5	29/10/18	7	8	Week 22	25/2/19	28	29
Week 6	5/11/18	9	10	Week 23	4/3/19	30	31
Week 7	12/11/18	11	12	Week 24	11/3/19	32	33
Week 8	19/11/18	13	14	Week 25	18/3/19	(none)	(none)
Week 9	26/11/18	15	16	Week 26	25/3/19	(none)	(none)
Week 10	3/12/18	17	18	EASTER			(none)
Week 11	10/12/18	19	(none)	Week 27	29/4/19	(none)	(none)
XMAS				Week 28	6/5/19	(none)	(none)
Week 15	7/1/18	(none)	(none)	Week 29	13/5/19	(exam)	(exam)

- Students are split into two groups, two lectures per week for each group.
- ’Week’ corresponds to the university week.

Lecture Titles and Lab/CW Synchronization (1/2)

Lecture	Lecturer	Lab	Description
1	JS/IP	L1	Course Introduction
2	JS/IP	L1	Applied Maths Overview
3	JS/IP	L1	Kinetic Theory ♯1
4	JS/IP	L1	Kinetic Theory ♯2
5	JS/IP	L1	Definition of a Fluid
6	JS/IP	L1	Change in Thermofluid Systems ♯1
7	JS/IP	L2	Change in Thermofluid Systems ♯2
8	JS/IP	L2	Cycles and Heat Engines
9	JS/IP	L2	Carnot Cycle: The Impossible Engine
10	JS/IP	L2	Practical Engine Cycles: Internal Combustion Engines
11	JS/IP	L3	Practical Engine Cycles: Gas Turbine Engines
12	JS/IP	L3	Conservation Laws
13	JS/IP	L3	Mass Conservation using a Control Volume
14	JS/IP	L3	An introduction to Momentum
15	LM		The hydrostatic equation
16	LM	L3	Hydrostatic force distributions
17	LM	L3	Bouyancy Forces
18	JS/IP	L3	Dimensional Analysis
19	JS/IP	L3	Buckingham Pi Theorem

Lecture Titles and Lab/CW Synchronization (2/2)

Lecture	Lecturer	Lab	Description
20	JS/DL		Flow Assumptions and Boundary Conditions
21	JS/DL		Convective and Diffusive Transport
22	JS/DL		Non Dimensional Convection-Diffusion
23	JS/DL		Flow Visualization methods
24	JS/DL		The Euler and the Bernoulli Equation
25	JS/DL		Force- Momentum Equation
26	JS/DL	L5	Actuator Disk Theory
27	JS/DL	L5	Steady Flow Energy Equation
28	JS/DL	L5	Practical Engine Cycles: Gas Turbine Engines (Revisited)
29	JS/DL	L5	Steady Mechanical Energy Equation
30	JS/DL	L6	Couette Flow
31	JS/DL	L6	Pipe Flows, viscous drag, turbulence
32	JS/DL	L6	Boundary Layers
33	JS/DL	L6	Separation and Pressure Drag

How to get the most out of this module (1/2)

Learning is hard. And we can't, won't, shouldn't, do it for you. *We will teach you how to learn.*

This module works as follows..

Lectures	Introduce, Define/Derive and Explain the Fundamental Principles.
Self Study	Follow through on the additional reading suggested in the Lectures.
Tutorial Classes	Be shown how to read, set up, solve and check workbook questions.
Self Study	Try additional workbook questions yourself.
1-2-1 time	Get help from your tutorial class academic when you get stuck (1-2-1 time).

You will get stuck. That's OK. Good even. But ASK for help. Please do not be afraid to ask. We want to help.

- We understand this place can be a big scary place for you. You're not top of the class any more.
- We understand that you have to adapt your learning. You are lot more autonomous here.
- We are mindful that many of you have come from different education systems and cultures.
- So it is really important that you let us know if you are having problems.

How to get the most out of this module (2/2)

This module is not easy because..

- It communicates knowledge through the language of *mathematics*
- Even with good mathematical fluency, the *conceptual ideas* need thought
- And this conceptual understanding is tested by solving problems

The tutorial classes are important because....

- Problem solving is a skill that cannot really be taught
- But most of all it requires practice. Lots of it.
- We help you get started in the tutorial classes - we go through the process with you.
- It is crucial you then practice on your own.

If you get stuck we'll help ! 1-2-1 time...

- 1-2-1 is dedicated time for you and the academic staff running your tutorial class. USE IT but *ask the right question*
- Wrong Question : 'I don't understand thermodynamics, please help'.
- Right Question : 'I can't do question X - I understand A,B,C, have tried XXX and YYY, what have I missed'.

Questions covered in Tutorial Sessions.

Tutorial sessions are offered to help students make the transition from solving maths to forming the problem mathematically.

These are taken directly from the workbook and the following long questions are suggested.

Semester 1	Section . Question	Semester 2	Section . Question
Week 1	(no tutorial)	Week 18	(Q+A)
Week 2	(ice cube)	Week 19	10.4,10.6
Week 3	3.3,3.9	Week 20	9.4,9.8
Week 4	3.13,3.15	Week 21	11.3,11.5
Week 5	5.2,5.4,5.6	Week 22	10.8
Week 6	6.3,6.7,6.8	Week 23	10.5,12.4
Week 7	7.4,7.8	Week 24	12.1(a),12.6
Week 8	9.6,9.14	Week 25	(Q+A)
Week 9	8.5,8.10,8.11	Week 26	(Q+A)
Week 10	4.1,4.2,4.3	EASTER	
Week 11	(Q+A)	Week 27	(Q+A)
XMAS		Week 28	(Q+A)
Week 15	(Q+A)	Week 29	(Q+A)

- GET INVOLVED. They are your chance to ask "why".
- Remember: Watching someone do the question does NOT mean YOU know how to do it. It will however help you know how to *approach* questions of the same type.
- In your first tutorial slot, you should discuss with your tutor a time/date that is fixed for that semester where the academic will be in his/her office for you.

A Wake Up Call (from a few years ago).

- The average mark for this module was 47% for UK students.
- The pass mark was 40%.
- Looking at the marks distribution, we see the expected "normal" distribution around the 50-60 range.
- There is a long tail to zero.
- You might say "but sir, the module is too hard".
- the Malaysian Students in the same year got an average of 71

Total Mark(%)	Count
0-9	5
10-19	23
20-29	13
30-39	21
40-49	39
50-59	88
60-69	105
70-79	48
80-89	19
90-100	10

The USMC students did all the workbook questions, and asked for more. The UK students complained they didn't have the solutions.

Take Personal Responsibility for your Learning.

Student Comments, having finished the year.

QUESTION: If you were to retake part I what improvements would you make based on your experiences in the previous academic year?

- "I would make sure I made a summary of all the notes from each week of lectures. This would make it much easier to revise in the summer."
- "I would certainly be more focused and organised in taking notes in lectures and reading up on things I didn’t understand as due to the hectic nature of the course of this was not done at the time it was left to the end of the year in the limited revision time. It was also very easy to fall into the trap of spending all study time on courseworks and revising for the weekly maths tests instead of actually learning the material as a constant process."
- "Start coursework earlier. Revise earlier (after each lecture)."
- "I would try and keep on top of tutorial sheets and things, and work a more 9-5 day, rather than being sporadic."

Summary of the Lecture

AIM : To learn how to solve problems in thermofluids sufficiently well to pass the module.

You now know..

- Broadly the scope of the module.
- How it is structured.
- The teaching materials we provide and where they are.
- The staff support you get in terms of lectures, tutorials, 1-2-1 time.

You know what we are going to do to support you through this module.

Lecture 2: Applied Maths Overview.

Last Lecture we covered:

- Organisation to the module.
- What we are going to do for you.
- What you need to do for yourself.

This lecture we are going to cover:

- An overview of all the mathematical tools required for this module.
- Designed to mention things *briefly*, some you will know already.
- The objective is when we are deriving thermofluid equations - you focus on understanding the physics rather than the maths.
- This is a heads up to let you know what is coming.

Detailed Lecture Content.

List of Material for Todays Lecture

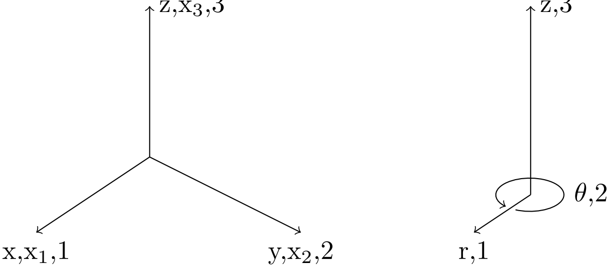
- Eulerian coordinate systems.
- Scalars and vectors. Points and fields.
- Simple vector operators.
- Finite and infinitesimal increments.
- Differentiation, gradients, total derivatives.
- Taylor series.
- Partial derivatives.
- Closed and open integrals.
- Integration in one dimension, surfaces, volumes.

We will probably go faster than you are used to today.
You can go back and watch this lecture again as many times as you need.

Eulerian Coordinate Systems.

- Coordinate systems measure things, here position-in-space
- Eulerian Coordinate systems define displacement relative to an origin.
- 'Displacement' therefore has two information components, 'how far' and 'what direction' (from the origin)
- The Coordinate axes define the directions of measurement.

The choice of coordinate system is a decision about how we measure 3D space.



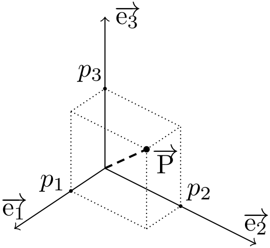
We will mainly use the Cartesian system, occasionally the Cylindrical Coordinate system.

Scalars and Vectors.

Information Representation

Scalars are an information "packet" that has one "bit" of info - magnitude : familiar examples are pressure, temperature.

Vectors are an information "packet" that has two "bits" of info - magnitude and direction. Each vector has 3 elements of information - one for each coordinate direction.



Example

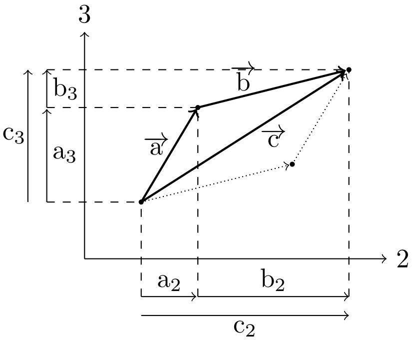
Position Vectors in a Cartesian Coordinate System - 3D displacement from the origin

- The position vector $\overrightarrow{P} = [x_1, x_2, x_3]$ or more generally $\overrightarrow{P} = [p_1, p_2, p_3]$.
- Gives direction and distance from the origin.
- $\overrightarrow{P} = p_1 \overrightarrow{e}_1 + p_2 \overrightarrow{e}_2 + p_3 \overrightarrow{e}_3$ - unit vectors defining the direction of each axis.
- $|\overrightarrow{P}| = (p_1^2 + p_2^2 + p_3^2)^{\frac{1}{2}}$ - the distance from the origin to $\overrightarrow{P}$ - the magnitude of p.
- If $|\overrightarrow{P}| = 1$ the vector is terms a unit vector, it has a *length* of 1 (unit).

Vector Operators (1/2) : Addition/subtraction/scalar multiplication.

We operate on numbers to produce other numbers

- + and - are same for scalar-scalar and vector-vector (no meaning for scalar-vector)
- * the same for scalar * vector. Different for vector*vector
- Division is not allowed by a vector



Addition/Subtraction:

- $\overrightarrow{a} + \overrightarrow{b} = \overrightarrow{b} + \overrightarrow{a} = \overrightarrow{c}$

Multiplication:

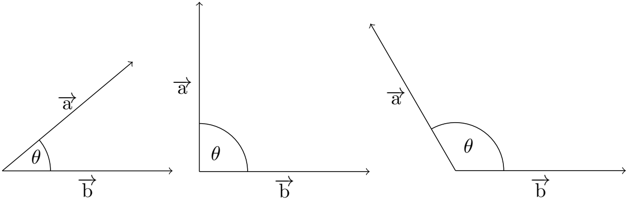
- $K(\overrightarrow{a} + \overrightarrow{b}) = K\overrightarrow{a} + K\overrightarrow{b}$

Example

- $\overrightarrow{P} = p_1\overrightarrow{e}_1 + p_2\overrightarrow{e}_2 + p_3\overrightarrow{e}_3,\ \overrightarrow{Q} = q_1\overrightarrow{e}_1 + q_2\overrightarrow{e}_2 + q_3\overrightarrow{e}_3$
- $\overrightarrow{R} = \overrightarrow{P} + \overrightarrow{Q} = (p_1 + q_1)\overrightarrow{e}_1 + (p_2 + q_2)\overrightarrow{e}_2 + (p_3 + q_3)\overrightarrow{e}_3$
- $\overrightarrow{R} = 2\overrightarrow{P} = 2p_1\overrightarrow{e}_1 + 2p_2\overrightarrow{e}_2 + 2p_3\overrightarrow{e}_3$

Vector Operators (2/2) : (inner) Dot Product.

The dot product of two vectors how much of one vector is in the direction of the other



$\vec{a} \cdot \vec{b} > 0$ $\vec{a} \cdot \vec{b} = 0$ $\vec{a} \cdot \vec{b} < 0$

$$\vec{a} \bullet \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\vec{a}||\vec{b}|cos\theta$$

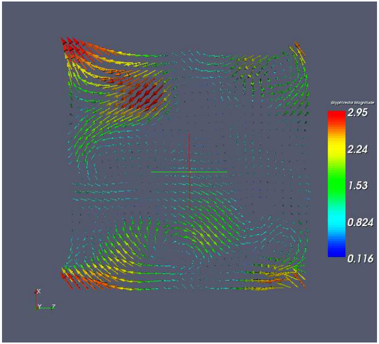
...because $\vec{e}_1 \bullet \vec{e}_1 = 1$ and $\vec{e}_1 \bullet \vec{e}_2 = 0$

$$(\vec{a} + \vec{b}) \bullet \vec{c} = \vec{a} \bullet \vec{c} + \vec{b} \bullet \vec{c}$$

Key uses in this module

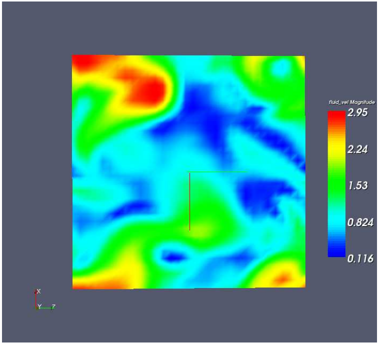
- Used to define force components in a given direction (eg $\vec{b}$ a unit vector in the required direction)
- Used also to define the mass flow through a surface when the flow is not normal to the surface (eg $\vec{b}$ a unit vector normal to the surface)

Scalar and Vector Fields.



(Vector) Velocity Field (ms^{-1}).

$\overrightarrow{U}$

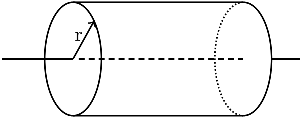


(Scalar) Velocity Magnitude Field (ms^{-1}).

$U = |\overrightarrow{U}| = (\overrightarrow{U} \bullet \overrightarrow{U})^{\frac{1}{2}}$

The Importance of *Gradients* In Thermofluids

The rate at which *information* changes in space and time defines the magnitude of the change



- Newtons Law $F = ma = m\frac{du}{dt}$.
- Pressure Gradients : $\frac{dp}{dx}$.
- Thermal Gradients : $\frac{dT}{dx}$.

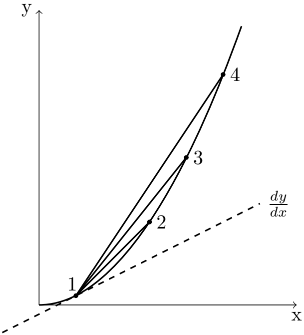
Gradients are incredibly important in thermofluids - they can be considered driving forces for things that happen.
The first order derivative is commonly known as a gradient.

Limits, Infinitesimal and finite increments.

Let us say we have a curve and we want to investigate the change in y with respect to the change in x.

The aim is to define how to represent a change in y as x nears x_1

- If all we are interested in is the finite change in y as x moves from 4 to 1, then Δy is the symbol used here. These are finite measurable changes.
- Sometimes we need to derive equations based on infinitesimal changes, here we would represent a change in y by δy .



- Looking at the graph, as point 4 moves from 4,3.. and on to 1 the change in δy the *gradient* gets more accurately represented.
- As $\delta y \rightarrow 0$ and $\delta x \rightarrow 0$, at point 1 (say) then $\frac{\delta y}{\delta x} \rightarrow \frac{dy}{dx}$ - the gradient of y with respect x at a point.
- More formally, $\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \frac{y(x+\delta x)-y(x)}{\delta x}$.
- This is known as a total derivative. The *change* in y is totally defined by the change in x.

Partial versus Total Derivatives.

The total differential change in $f(x, y)$ when x and y change.

Consider a *small* part of the surface, where f changes by δf and x and y by δx and δy

$$\delta f = f(x+\delta x, y+\delta y) - f(x, y)$$

$$\delta f = \left[\frac{f(x+\delta x, y+\delta y) - f(x, y+\delta y)}{\delta x}\right]\delta x + \left[\frac{f(x, y+\delta y) - f(x, y)}{\delta y}\right]\delta y$$

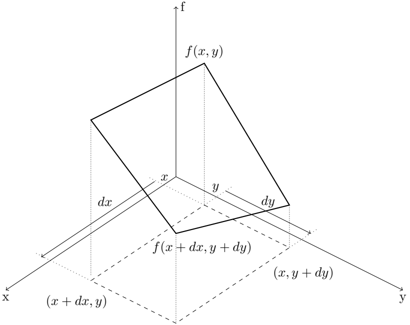
As $\delta y \to 0$ and $\delta x \to 0$ then

$$df \to \left[\frac{\partial f}{\partial x}\right]dx + \left[\frac{\partial f}{\partial y}\right]dy.$$

$\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are the partial derivatives of f with respect to x and y.

The former is at constant y and the latter is at constant x.

These are used as definitions - you will not be asked to manipulate these in equations (but I might!).



Taylor Series.

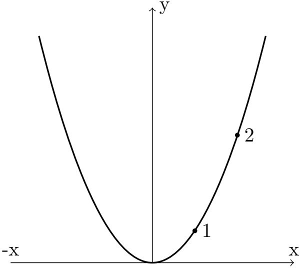
A Taylor series uses information, including first and higher derivatives at one point, to estimate information at another point.

$$y(x_2) = y(x_1) + (x_2 - x_1)\left.\frac{dy}{dx}\right|_{x_1} + \frac{(x_2-x_1)^2}{2!}\left.\frac{d^2y}{dx^2}\right|_{x_1} \cdots \frac{(x_2-x_1)^n}{n!}\left.\frac{d^ny}{dx^n}\right|_{x_1}$$

In theory accuracy improves with the order used.
However this *assumes* we have accurate estimates of higher order terms.

We use Taylor series to derive equations, and in this case we take a very small interval.
Because of this we can use only a first order approximation

$$y(x_2) = y(x_1) + (x_2 - x_1)\left.\frac{dy}{dx}\right|_{x_1}$$

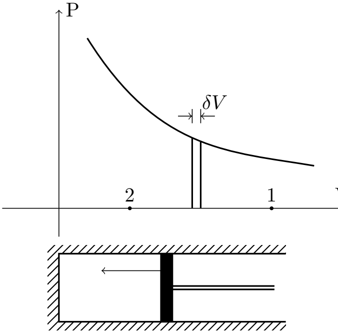


Example

- Pressure varies in the x direction as $p(x) = 4 + 2x^2$. Given the pressure at $x = 3$ is 22 Pa, what is the pressure at x=3.1?
- $(x_2 - x_1) = 0.1$, $\frac{dp}{dx} = 4x$. So $\left.\frac{dp}{dx}\right|_{x=3} = 12$
- $p(x = 3.1) = p(x = 3) + 0.1 * \left.\frac{dp}{dx}\right|_{x=3} = 22 + 0.1 * 12 = 23.6$

The Importance of Integration in Thermofluids

Integration allows us to calculate a finite change from many infinitesimal changes



To provide some meaning consider a piston compressing a gas.

- Energy = Force × distance.
- Pressure = Force/Area.
- Energy = Pressure × Area × distance.

$\delta W = pA\delta x = p\delta V$

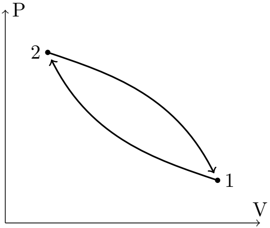
- An infinitesimal change in work (energy) is required to change the volume of a gas at a given pressure.

The integral effect of these infinitesimal changes results in $W = \int \delta W = p_1 A\delta x_1 + p_2 A\delta x_2 .. + p_n A\delta x_n = \int pAdx$.

- As written above, this is an indefinite integral - if you integrate it a constant arises which has to be defined from a boundary condition.
- The alternative is to use definite integrals, e.g. $W = \int_{V_1}^{V_2} pdV$ - this is however useless for deriving equations and the former method is preferred.

Cycle integrals.

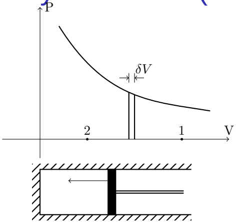
When considering the motion of a piston in a cylinder, it performs a cycle, starting at one point and returning that point. Plotting how pressure and temperature change in the cylinder ensures that the cycle is closed - it ends where it starts.



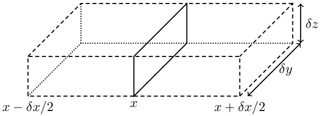
This has a special notation

$$W_{net} = A\int_1^2 pdx + A\int_2^1 pdx = A\oint pdx.$$

How you will use (or not) 1, 2 and 3D integration.



1D Integral : Change in x only
$Area = W = \int dW = \int pAdx$



2d Integral : Change in y and z here
$\dot{V} = \int U(y,z)dA = \int \int U(y,z)dydz$

Ways in which you will use 1,2,3D Integration

1D : You will be asked to define and integrate 1D integrals.
2D : You will be asked to recognize 2D integrals. You will always be able to simply them to 1D to integrate them.
3D : integrals are used for definitions only. $m = \int \rho dV$. If incompressible, then $m = \rho \Delta V$

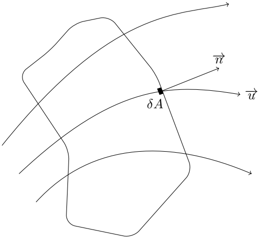
Example

- You are asked to integrate $\dot{V} = \int U(y,z)dA$, assuming the width of the channel in the y direction is ΔY, and the velocity does not vary in the y direction.
- This means that we can express $dA = \Delta Y dz$ and that $U(y,z)dA = U(z)\Delta Y dz$ or $\Delta Y U(z)dz$. Therefore $\dot{V} = \int U(y,z)dA = \Delta Y \int U(z)dz$.

Working out the net mass flux through a fixed volume

Example

- To cement this knowledge, we will carry out an example of using length nomenclature, vectors and integration operations.



we choose a infinitesimal area (δA)

- Velocity through δA is $\overrightarrow{u}$
- the normal component of that velocity is $\overrightarrow{u} \bullet \overrightarrow{n}$
- the infinitesimal mass flow is $\delta \dot{m} = \rho(\overrightarrow{u} \bullet \overrightarrow{n})\delta A$
- we sum up all our $\delta \dot{m}$s, eg $\dot{m} = \int \rho(\overrightarrow{u} \bullet \overrightarrow{n})dA$

For an *incompressible* fluid, the amount of mass going into the fixed volume must equal the amount coming out thus,
$\int \rho(\overrightarrow{u} \bullet \overrightarrow{n})dA = 0$
Is the conservation of mass equation for an incompressible fluid.

Summary, Further Study, Homework

Lecture Summary

- Eulerian coordinate systems.
- Scalar and vector points and fields.
- Simple Vector operators.
- Finite and infinitesimal increments.
- differentiation, gradients, Partial and total derivatives.
- Taylor series.
- The meaning of $\delta, \partial, d, \Delta$.
- Closed and open integrals.
- Integration in one dimension, surfaces, volumes.

Suggested Further Reading

- Crowe et al., Chapter 5.
- Shrimpton, Section 2.1.

Suggested Further Study

- Try chapter 2.

Lecture 3: Kinetic Theory

Last Lecture we covered:

- Eulerian coordinate systems.
- Scalar and vector points and fields.
- Simple Vector operators.
- Finite and infinitesimal increments. The meaning of $\delta, \partial, d, \Delta$.
- differentiation, gradients, Partial and total derivatives.
- Taylor series.
- Integration in one dimension, surfaces, volumes.

This Lecture we are going to cover:

- Historical context in understanding of gases.
- The Kelvin Scale.
- Microscopic (molecular) motion in a gas.
- Macroscopic (i.e. pressure, temperature) quantities.
- Specific heats, Internal Energy, Enthalpy and Work.
- First Law of thermodynamics.

Lecture Warning : This analysis is for a very simple gas. The relationships $C_p = 5R/2, C_v = 3R/2$ do not apply for 'normal' gases, like air.

Brief history of theory and application in thermodynamics.

- In 1738 Daniel Bernoulli proposed that gases consist of great number of molecules moving in all directions. Their impact on a surface causes the pressure that we feel. What we experience as heat is simply the kinetic energy of their motion. At this time heat was regarded as a sort of weightless and invisible "calorific" fluid that flowed when out of equilibrium.
- Sadi Carnot, aged 28, in 1824 abstracted the essential features of the steam engine. This resulted in a model thermodynamic system upon which exact calculations could be made. Carnot knew that the conduction of heat between bodies at different temperatures is a wasteful and irreversible process, which must be eliminated if the heat engine is to achieve maximum efficiency.
- In 1845, James Joule reported his best-known experiment, involving the use of a falling weight, in which gravity does the mechanical work, to spin a paddle-wheel in an insulated barrel of water which increased the temperature.
- James Maxwell (1873) formulated key parts of "kinetic theory", which gave the proportion of molecules having a certain velocity in a specific range and enabled macroscopic properties to be defined from molecular motion.
- Nikolaus Otto developed the spark ignition engine in 1876. Rudolf Diesel understood the Carnot cycle, in 1893 defined the compression ignition engine.
- Until the 1900's atoms were considered purely hypothetical constructs. An important turning point was Einstein's (1905) work. Made measurable predictions based on the assumed motion of molecules in gases.

Early Work on the Nature of Gases (Gas Law and Two Point Temperature scales).

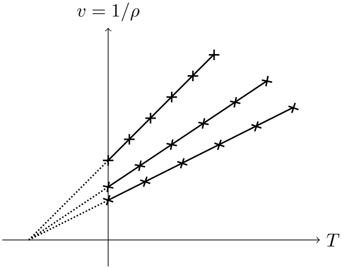
- Boyle's law (≈ 1662) found that as long as temperature is constant the product of absolute pressure and volume is constant when a gas is expanded: $pv = \frac{p}{\rho} = const.$
- Charle's law (≈ 1780) found as long as the pressure stays at a constant gas volume and temperature are directly proportional $\frac{T}{v} = \rho T = const.$
- Avagadro (≈ 1811) found out a certain volume of gas at a certain T, P held a certain number of molecules, $N_A = 6 \times 10^{23}/mol$.
- Clapeyron combined Charles's law with Boyle's law (1834) to produce a single statement which would become known as the *ideal* gas law $\frac{p}{\rho T} = const.$

No-one knew what heat (energy) actually was, or that molecules existed. They all measured temperature by various two-point methods.

- Amontons (≈ 1700) discovered that the pressure of a fixed mass of gas kept at a constant volume is proportional to the temperature. Amontons discovered this while building an "air thermometer".
- Fahrenheit's mercury thermometer defined a scale 180 points (degrees) between the freezing point of a brine mixture and the boiling point of water.

In these *two point* methods the measurement of temperature is relative to some datum. There is no physical meaning in these scales !

Lord Kelvin and the one-point temperature scale.



Kelvin noticed..

- These are linear for a reasonable range of $V - T$ space.
- If you extrapolate these to lower T they all cross the $V = 0$ axis at the same point.
- In theory this means that all gases, at some *fundamental* Temperature, have zero volume.

Kelvin supposed..

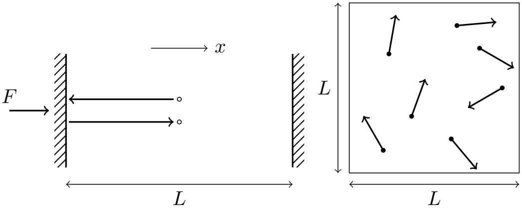
- If a gas volume is defined by the motion of gas molecules, then this temperature defines when molecules have zero motion.
- If molecules has zero motion, then a gas comprising such molecules also has zero internal energy.
- Therefore the Kelvin temperature scale might be related directly to the gas internal energy.
- The Kelvin scale is related to the Centigrade Scale by $T(K) = T(degC) + 273.15$.

Note : Sometimes you have to use K scale temperatures, and sometimes it does not matter. It is best to always use K scale temperatures in your calculations.

Setting up our microscopic molecular system/one particle dynamics.

Consider a cubic box of side L and volume V containing a gas

- The simple low pressure gas at T, P.
- Each of the N molecules has a mass m_a and velocity components u_x, u_y, u_z.



Simple case first : One molecule, moving in the x-direction only

- The impact of that *one* molecule on *one* wall of the box normal to that direction.
- Particle Mom (Ns) change with LH wall = mom before - mom after = $-m_a u_x - (m_a u_x) = -2m_a u_x$.
- Therefore force required to stop the wall moving is $2m_a u_x$.
- Number of times per second the wall is hit by this particle is $\frac{|u_x|}{2L}$.
- The momentum ($kgms^{-1}$) change per second (N) on one wall $= \frac{m_a u_x^2}{L}$.

Therefore the pressure on the wall, due to this one particle is: $p = \frac{F}{A} = \frac{m_a u_x^2}{LA} = \frac{m_a u_x^2}{V}$.

Pressure due to all the particles: moving in all directions.

In reality each particle is moving in three directions and
$u^2 = u_x^2 + u_y^2 + u_z^2$

- u is the *speed* of the particle. If we sum over all the N particles in the box :
- $\sum_{n=1}^{N} u_n^2 = \sum_{n=1}^{N} (u_{x,n}^2 + u_{y,n}^2 + u_{z,n}^2) = \sum_{n=1}^{N} u_{x,n}^2 + \sum_{n=1}^{N} u_{y,n}^2 + \sum_{n=1}^{N} u_{z,n}^2.$
- There is no preferred direction : $\sum_{n=1}^{N} u_{x,n}^2 = \sum_{n=1}^{N} u_{y,n}^2 = \sum_{n=1}^{N} u_{z,n}^2$ and..
- $\sum_{n=1}^{N} u_n^2 = 3 \sum_{n=1}^{N} u_{x,n}^2,$

we define the mean KE of the particle speeds as:
$\bar{u^2} = \frac{1}{N} \sum_{n=1}^{N} u_n^2 = \frac{3}{N} \sum_{n=1}^{N} u_x^2$

- Remember the pressure on the wall, due to one particle is: $p = \frac{m_a u_x^2}{V}$.
- So the pressure on one of the walls, from all the particles, moving in all directions is: $p = \frac{2N}{3V} \left(\frac{1}{2} m_a \bar{u^2} \right)$.
- So, we have the pressure-volume product equal to the mean kinetic energy of the N particles in our box:
- $pV = \frac{2N}{3} \left(\frac{1}{2} m_a \bar{u^2} \right)$.

Pressure, Volume, Temperature and Internal energy, per mole.

Relating mean molecular kinetic energy to Temperature

Using a single macroscopic property, temperature to represent the integral effect of many molecular velocities.

$\frac{1}{2} m_a \bar{u^2} \alpha T$

The proportionality constant (Boltzmann constant), k_B (JK^{-1}) relates molecular kinetic energy (J) to Kelvin (K)

$\frac{1}{2} m_a \bar{u^2} = \frac{3}{2} k_B T$

... via $pV = \frac{2N}{3}\left(\frac{1}{2} m_a \bar{u^2}\right)$ gives a form of the gas law....

$pV = Nk_B T$.

This tells us the pressure-volume product is a function of T

- If the box volume and contents stays the same, the pressure rises linearly with T.
- This is exactly what Amontons discovered while building an "air thermometer" in ≈ 1700, but did not know why.

Pressure, Volume, Temperature and Internal energy, per mass.

Finally we may relate the Boltzmann constant to the Universal Gas Constant ($J/molK$) using Avagadros Number ($/mol$) $k_B = \frac{R_u}{N_A}$ and we recover the familiar gas law.

$pV = nR_uT$ where $n = \frac{N}{N_A}$ is the mol of gas present.

Starting from the gas law per mole: $pV = nR_uT$: $R_u = 8.314kJ/kmolK$.

Multiply by the molar mass of the gas: $m_u = \frac{m}{n}$: $pV = m_u n \frac{R_u}{m_u} T = mRT$.

Here R is the mass gas constant (J/KgK). This IS dependent on the type of gas since: $R = \frac{R_u}{m_u}$.

Finally, dividing both sides of the gas law gives the intensive form of the equation: $p(V/m) = RT$ to $pv = \frac{p}{\rho} = RT$.

This is known as the "ideal gas equation" and is a very good approximation for low pressure gases. In this module, if you need to use the gas law, you will use this.

Gases that follow this law are known as "ideal gases".

Gas Heating in a Constant volume (Gas internal energy).

Our box is a constant volume device, if we heated the walls and the molecules picked up (microscopic) kinetic energy over time the macroscopic energy measure (Kelvin temperature) would rise. Internal energy of a fluid is defined by the specific heat at constant volume, e.g.

$E_u = N\frac{1}{2}m_a\bar{u}^2 = \frac{3}{2}Nk_BT = \frac{3}{2}nR_uT = \frac{3}{2}mRT = mC_VT$. As a specific quantity, $e_u = C_VT$.

Therefore: $C_V = \frac{3}{2}R$. only true for atoms

Formally, C_V is defined as the energy required to raise 1 kg (not 1 mol) of gas by 1 K at constant volume.

$$C_V = \left(\frac{\partial e_u}{\partial T}\right)_V.$$

Note: It is the specific heat property that defines the specific energy contained in a material - and is a material property. As we will see later the energy is a process property.

In reality the specific heats are a function of temperature. We will assume they are constant in this module.

An ideal gas that has constant specific heats is known as a "perfect gas".

Gas Specific Heat at Constant Pressure (Gas Enthalpy).

For the constant pressure specific heat, we now imagine our box expands in one direction when the walls are heated and the molecules pick up speed (as above) to keep the pressure constant.

Here the gas molecules are using up some of their energy to move the box out as well as moving faster. The energy expended in moving one face of the box out by δL is: $\delta W = F\delta L = pA\delta L = p\delta V$.

From the Gas Law, at constant pressure, $pV = mRT$, $W = p\delta V = mR\delta T$.

So, energy added at constant pressure is $E_h = mC_V\delta T + mR\delta T = mC_p\delta T$ or as a specific quantity, $\delta e_h = C_p\delta T$. Since $C_V = \frac{3R}{2}$, $C_p = \frac{5R}{2}$. Only true for atoms.

This is the gas enthalpy, and is the sum of the internal energy of the gas and the energy required to increase the volume of the gas at constant pressure by a given temperature.

Formally, C_p is defined as the energy required to raise 1 kg (not 1 mol) of gas by 1 K at constant pressure.

$$C_p = \left(\frac{\partial e_h}{\partial T}\right)_P.$$

Enthalpy, Internal Energy, and the First Law of Thermodynamics.

$C_V = \left(\frac{\partial e_u}{\partial T}\right)_V$ or $e_{u,2} - e_{u,1} = e_{u,12} = \int_{T_1}^{T_2} C_V dT = C_V(T_2 - T_1)$.

- Heat Transfer per unit mass of any process at constant volume.
- This only increases the temperature.

$C_P = \left(\frac{\partial e_h}{\partial T}\right)_P$ or $e_{h,2} - e_{h,1} = e_{h,12} = \int_{T_1}^{T_2} C_P dT = C_P(T_2 - T_1)$.

- Heat Transfer per unit mass of any process at constant pressure.
- This increases the temperature and does some work on the environment.

$E_h = mC_V\delta T + mR\delta T = mC_P\delta T$, and that $\delta W = p\delta V = mR\delta T$.

- Therefore $e_{h,12} = e_{u,12} + w_{12}$: Note we can define specific work in terms of specific volume.
- Specific heats and the mass Gas constant: $C_P = C_V + R$.
- Another useful parameter is the ratio of Specific Heats $\gamma = \frac{C_P}{C_V}$.

So if we have a heat transfer into a mass (a system) : $Q_{12} = E_{u,12} + W_{12}$.

- This is the First Law of Thermodynamics - in specific terms $q_{12} - w_{12} = e_{u,12}$.

Summary and Suggested Reading

Lecture Summary

- Historical context in understanding of gases.
- The Kelvin Scale.
- Microscopic (molecular) motion in a gas.
- Macroscopic (i.e. pressure, temperature) quantities.
- Specific heats, Internal Energy, Enthalpy and Work.
- First Law of thermodynamics.

Suggested Further Reading

- Cengel and Boles, Sections 1.8.
- Fenn JB, Engines, Energy, Entropy, WH Freeman and Company, San Francisco, USA, 1982.
- Shrimpton, Section 2.7.

Suggested Further Study

- Try chapter 3 questions.

Lecture 4: Kinetic Theory (2/2)

Last Lecture we covered:

- Historical context in the understanding of gases.
- The Kelvin Scale.
- Microscopic (molecular) motion in a gas.
- Macroscopic (i.e. pressure, temperature) quantities.
- Specific heats, Internal Energy, Enthalpy and Work.
- First Law of thermodynamics.

This Lecture we are going to cover :

- Why our molecular description of a gas is incomplete or:
- Why temperature, pressure, first law does not completely define our gas.
- Introduction to entropy from a microscopic point of view.
- Maxwell's Demon and the range of states.
- Example of entropy change with no other change.
- 3^{rd} Law of Thermodynamics.

Variables known thus far.

We have some state variables that define the state of a gas.

- For a gas, P, T and V (*extensive* form) or v (*intensive* form) define the state of a gas. They are known as state variables.
- We are missing one (be examined next).

We have some universal constants, that apply to all gases.

- Boltzmann constant k_B: relates (microscopic) molecular kinetic energy to (macroscopic) temperature.
- Universal Gas Constant R_u - relates the (microscopic) number of molecules to the (macroscopic) energy they contain.

We have some material constants, that define the capacity of the gas to change state:

- C_p, C_v, R, γ
- remember $C_p = 5R/2, C_v = 3R/2, R$ only applies to noble gases.

However our description of our microscopic molecules in a box is incomplete: our description of energy requires a measure of its quality as well as its magnitude: entropy.

Why our macroscopic description of the molecules in a box system is incomplete.

The problem is the maths doesn’t fit with reality :

- Gases will tend to expand into any available space.
- Hot gases tend to transfer heat to cooler gases.

We need a variable to define this *directionality* of real life. It is called entropy.

1. Energy is well behaved conserved quantity we can develop precise conservation laws for.
2. Entropy cannot be defined precisely, is not conserved and at best, stays constant.

There are far reaching implications of entropy generation in engineering that we will discover in this course:

Example

- The inability to convert all heat energy to work.
- The loss of energy in fluid systems due to friction/other irreversible losses.
- If we mix two gases together, of two different temperatures, the total energy does not change, but the entropy increases - and the "energy quality" has reduced, irreversibly.

We need to define energy quality and well as quantity

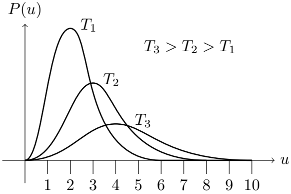
The molecular speed distribution of our molecules in our box as a function of T.

Temperature represents the mean kinetic energy of the molecules in our box

It turns out that we can work out exactly the probable speed distribution of the molecules.
It is known as the Boltzmann distribution.

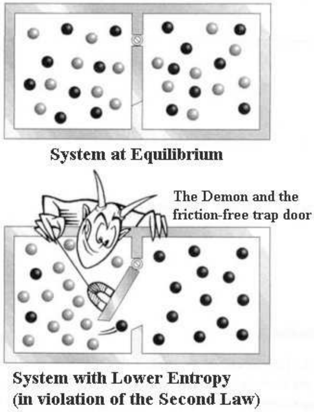
It has the following properties:

- It has a high velocity tail : There is also a low probability of very high speed molecules present.
- Higher energy distributions have a *wider range* of possible velocities and because of this...
- The probability that any one molecule will have a certain velocity is lower for higher energy distributions.



It is this *range of possible states* that is the key to understanding entropy at a microscopic level.

Why we cannot define entropy exactly



System at Equilibrium

The Demon and the friction-free trap door

System with Lower Entropy (in violation of the Second Law)

Maxwell explained the inexactness of entropy as follows.

Imagine two compartments with a tiny hole. Connecting them containing gas at the same temperature.

He then imagined a demon doorkeeper, letting through the fast molecules into one compartment, and slow ones through into the other.

Over time, one compartment would heat up and one would cool down.

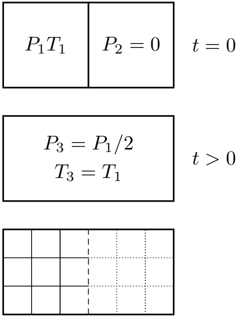
Over short times this might actually happen, but over long times it never will.

Key point is you can never prove this by considering a single or a few particles. You have to consider *many* particles, over a *sufficiently long time* and claim that reality is actually only very very likely.

The molecular speed distribution of our molecules in a box as a function of V.

Example

Unresisted Gas Expansion



- Consider molecules in one half of the box, where the other half is an empty vacuum.
- The partition is removed and the molecules over time fill the box. How has the state of the gas changed?
- The gas has expanded against a vacuum so no work has been done. Speed distribution has not changed.
- The box is insulated, there is no heat transfer and the molecules have lost no internal energy.
- Therefore the energy of the molecules has not changed.
- The quality of the energy has however has degraded because of the loss of pressure.

In terms of microscopic "range of possible states" each molecule has more *position* states after the partition is removed. If we divide each section into 9 areas, at $t = 0$ the probability of finding a molecule in any one area is 1 in 9, after it is 1 in 18. This increases the molecular entropy.

Definition of microscopic (statistical) entropy.

A definition of microscopic (and statistical) entropy

$S = k_B ln N_S$ where N_S is the number of possible states.

Example

- Previously, the two entropies are measured as $S_1 = k_B ln 9$ and $S_2 = k_B ln 18$
- The change however is $\Delta S = k_B ln 2$

- When gases of two different temperatures mix in the same volume.
- When a gas expands in an unresisted manner to increase volume.

- The microscopic "range of possible states" therefore has two forms of "state": velocity and position.

This completes the description of an ideal gas P, V, T, S. In a few lectures we apply this to real engine efficiency.

3^{rd} Law of Thermodynamics.

The definition of $S = k_B ln N_S$ also gives us a fundamental reference point for gases.

- As the temperature of the gas tends to absolute zero then so does the entropy.
- This can be understood in terms of the mean thermal speed of the molecules all tending to a zero state.
- Then, the molecules all have the same (zero) speed, and hence all have the same state. In other words the probability of the molecules having zero speed is 1.
- Likewise the gas theoretically occupies zero volume, and therefore the position of the molecules are all known, Therefore the number of possible position states a molecule might have is also 1.
- Therefore in velocity-position space, there is only one molecular state at absolute zero, therefore $N_S = 1$ and $S = k_B ln N_S = 0$.
- This is sometimes known as the Third Law of Thermodynamics. It gives a further insight into the Kelvin temperature scale.

Summary and Suggested Reading

Lecture Summary

- Why our molecular description of a gas is incomplete or:
- Why temperature, pressure, first law does not completely define our gas.
- Introduction to entropy from a microscopic point of view.
- Maxwell's Demon and the range of states.
- Example of entropy change with no other change.
- 3rd Law of Thermodynamics.

Suggested Further Reading

- Cengel and Boles, Sections 1.8.
- Fenn JB, Engines, Energy, Entropy, WH Freeman and Company, San Francisco, USA, 1982.
- Shrimpton, Sections 2.8.

Suggested Further Study

- Can complete chapter 3 questions, worked solution 3.16

Lecture 5: Definition of a Thermofluid

Last Lecture we covered:

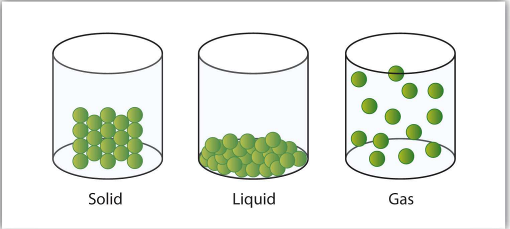
- Why our molecular description of a gas is incomplete or:
- Why temperature, pressure, first law does not completely define our gas.
- Introduction to entropy from a microscopic point of view.
- Maxwell’s Demon and the range of states.
- Example of entropy change with no other change.
- 3rd Law of Thermodynamics.

This lecture we are going to cover:

- The difference between a solid and a fluid.
- A fundamental look at what a fluid is from a microscale (molecular) viewpoint, using the simplest fluid - a pure low pressure gas.
- Relationship between the microscale and the macroscale.
- The origin of key fluid properties seen on the macroscale.
- Examination of pressure as an example of microscale/macroscale perspective.

Solids and Fluids (Liquids and Gases).

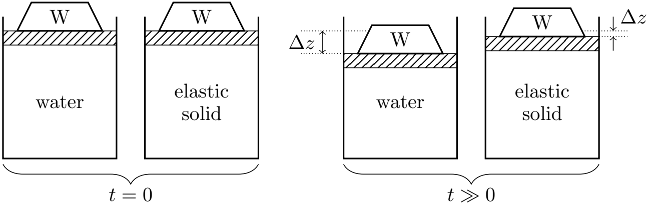
- Solids have molecules that do not move relative to one another, and the molecules are very close together and have strong attractive forces between them.
- Liquids have molecules close enough together for attractive forces to dominate and keep a given mass occupying a given volume, but has no preferred shape.
- Gases have molecules far apart and they have no attractive forces between them and expand to fill any available volume, and has no preferred shape.



Response of Solids and Fluids to a Normal Stress.

A weight on a piston acting in the direction of gravity in two cylinders

For a normal stress, one containing a fluid and one containing an elastic solid . In both cases the molecules of the material next to the wall stay in contact with their respective molecules of the wall material.



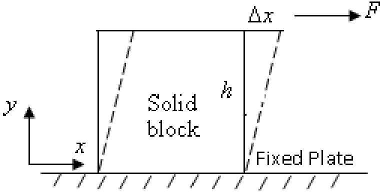
Conclusion

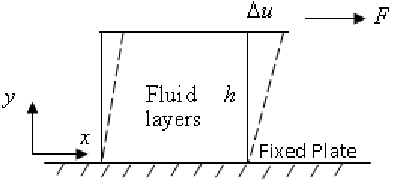
Solids and fluids behave in a similar way when subjected to a normal force (more specifically a normal stress (force per unit area).
A *compressive* force in the fluid/solid balances the weight on the piston and this is the *equilibrium* state, where *nothing* is moving.

Response of Solids and Fluids to a Shear Stress.

Two flat large plates separated by a narrow gap

In one case the gap is separated by a viscous fluid, treacle say. The other is our elastic solid. We now apply a shear force.





The solid behaves *as before*: gives a little and then acquires a new static equilibrium, with the stress in the deformed solid balancing the shear force created by the plate motion.

The fluid however allows the plate to move continuously as long as the shear force is applied and the equilibrium state is where the plate and the fluid are in *continuous* motion. The motion stops only when the force applied to the plates ends.

Conclusion

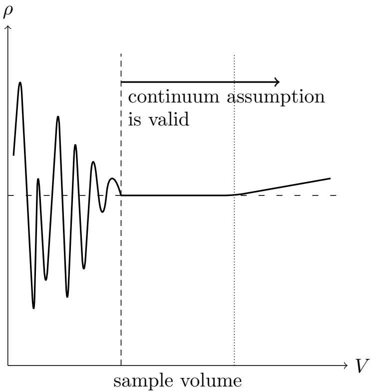
Solids and fluids behave *differently* under shear. Shear stresses in the fluid are ONLY present when the fluid is in motion.

Continuum Assumption : A "fluid" is an approximation.

Macroscopic average molecular measures

If a molecule weighs a mass M kg, then if there were N molecules in a box of volume Vm^3 then the density of the gas would be $\rho = \frac{NM}{V}$
In defining a density (or any other fluid property, like pressure, temperature, viscosity, thermal conductivity) we have made a continuum approximation and we have assumed our box is the "right size".

- If our box was too small, we might not count enough molecules to get a good average.
- If our box is too big we might not see a macroscopic change in a fluid property e.g. temperature/density - all our molecules are not the same on average.



What we are assuming is that if our box is the right size then we have a continuous material whose average properties (say density) can change smoothly from place to place. In reality molecules carry the information, and communicate this by undergoing many collisions.

Lecture 5: Examples of continuum approximations: using gases.

Let us assume we are interested in changes in some property (temperature say) in length scales of ≈1 mm, so we need a box of side 1 mm.

First let us assume that we have enough molecules at atmospheric temperature and pressure at ground level, therefore:

- $\rho = \frac{p}{RT}$.
- $T = 300K$, $R = 287J/kgK$, $V = 10^{-9}m^3$, $p = 10^5 Nm^{-2}$: $m \approx 10^{-9}kg$.

So, if we decided to work at an altitude of 10 km ($p \approx 0.26.10^5 Nm^{-2}$, $T = 220K$), $V = 2.4.10^{-9}m^3$, L = 1.33 mm:

- The length scale we can resolve increases slightly.

Or, if we wanted to examine the flow in a microchannel (≈10 microns)...

- Our continuum assumption fails - and all of the thermofluids you learn is this degree no longer applies!!

Generally, the length scales for liquids are not a problem, and also not a problem most of the time for gases.

The remainder of ALL thermo and fluids knowledge taught in your degree is predicated on valid assumption here !

Density (Mass per unit volume) and Pressure Definitions

Density Variations

- $\rho = \frac{m}{V}$. Sometimes we also use specific volume: $v = \frac{1}{\rho} = \frac{V}{m}$.
- Often it is defined relative that of water and is (very unfortunately!) called specific gravity. $SG = \frac{\rho_{liquid}}{\rho_{water}}$.
- Gases have their density defined by the gas law $\rho = \frac{p}{RT}$

Pressure Variations

- Note p in the above equation is the absolute pressure, i.e. the pressure relative to a perfect vacuum.
- Usually what we measure or use the gauge pressure, which is the pressure relative to some datum, usually atmospheric pressure, $p_{abs} = p_{gauge} + p_{atm}$
- Pressure in SI units is N/m^2 or Pa. Sometimes it is referred to in bar ($10^5 Pa$)

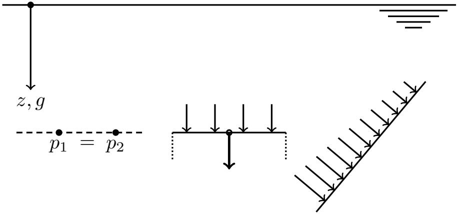
Temperature

- In the gas law, must use K !

How is pressure used, what is it?

Pressure can be source of much confusion

You need to be *aware* of the context of it's use. In the last few slides we have seen several examples of its use.



Pressure has units of N/m^2

- $\rho = \frac{p}{RT}$: Here it is absolute scalar property of the fluid, and defines the gas density along with the gas temperature.
- $p_{abs} = p_{gauge} + p_{atm}$: It can be defined in both relative and absolute terms. The key is if the fluid can be assumed to be incompressible.
- Hydrostatic Pressure. It is often termed a "normal stress", which is a vector quantity - a force per unit area. This is not true since pressure is a scalar : it has a magnitude but no direction.
- The direction of the force per unit area (stress) termed "pressure" is defined by the normal of the area through which the pressure is acting, e.g. $F = pA$.
- Be careful: Many people say "force due to pressure...", which is only $\frac{1}{2}$ the information.

Summary and Suggested Reading

Lecture Summary

- The difference between a solid and a fluid.
- The Continuum Assumption
- Relationship between the microscale and the macroscale.
- The origin of key fluid properties seen on the macroscale.
- Examination of pressure as an example of microscale/macroscale perspective.

Suggested Further Reading

- Crowe et al. Section 3.1-3.2.
- Shrimpton, Section 2.5.

Suggested Further Study

- try chapter 5.

Lecture 6: Change in Thermofluid Systems (1/2)

Last Lecture we covered:

- The difference between a solid and a fluid.
- The Continuum Assumption
- Relationship between the microscale and the macroscale.
- The origin of key fluid properties seen on the macroscale.
- Examination of pressure as an example of microscale/macroscale perspective.

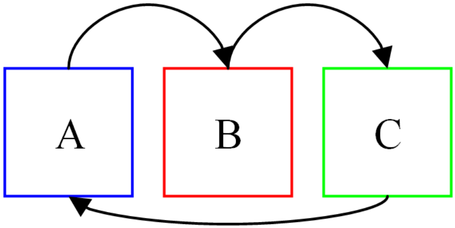
This lecture we are going to cover:

- Equilibrium and the Zeroth Law of thermodynamics.
- States, Processes, paths.
- Quasi-equilibrium assumption for processes.
- Non-equilibrium processes and Initial/final states.
- Reversible and irreversible processes.
- Displacement Work.
- Heat and work as Process properties.
- P-V diagrams.
- Types of Processes, paths.

Zeroth Law of Thermodynamics.

Some theoretical book-keeping..

....that implicit equilibrium across systems through the environment.



which implies...

If system A is in thermal equilibrium with the environment and system B is in thermal equilibrium with the environment then system A and B are also in thermal equilibrium.

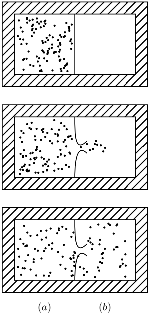
Internal Equilibrium, Two Property Rule

Defining (constraining) change so we can measure it

If a system is isolated from its environment and nothing is changing the system (a gas in this case) is said to be in internal equilibrium.

Take the case of rigid (no work transfer) insulated (no heat transfer) box containing two compartments (a) containing a gas and (b) a vacuum, separated by a diaphragm.

- System in internal equilibrium in section (a). Nothing in section (b).
- Diaphragm breaks - gas properties (p, T) vary in space and time, system is not in internal equilibrium.
- System again in a new state of internal equilibrium.



here is a state of *dynamic* equilibrium between the two sections

Internal equilibrium is a state whereby a fluid property may be defined by a single value for that system. When a system is in equilibrium - the two property rule is valid.

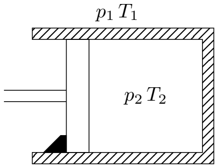
Example

If we know T and P of a gas, then we know everything, and can always work out ρ (using the gas law). S relationships to come..

Other Forms of Equilibrium between System and Environment

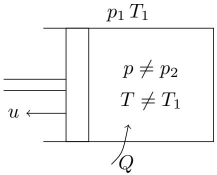
Start : a cylinder with a stop

The cylinder gas (*system*) is in internal equilibrium since nothing is in motion and no thermal contact to the environment.



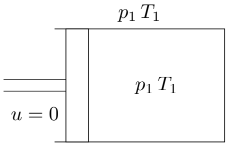
The cylinder insulation is removed and the stop taken away

The piston starts moving and also heat transfers through the cylinder walls. Neither internal, mechanical nor thermal equilibrium is valid here.



The piston motion stops as does the net heat flow across the cylinder wall

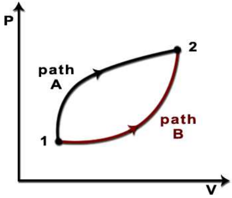
The system is in mechanical equilibrium (no change in the rate of piston motion over time) thermal equilibrium (zero net heat transfer across the cylinder wall) and internal equilibrium (uniform conditions in the system)



Terminology : Initial/final states : Process and Path.

Again, trying to *describe* change *quantitatively*

- state : A gas in equilibrium.
- process : doing something to a gas to change its state.
- initial/final : (states) ..of a process
- path : The trajectory in state-space by the process.



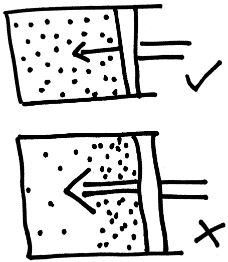
Points to note :

- A 2D plot fully describes the process, because of the two property rule.
- To specify the process (and define the path) the initial and final states and the transfers across the system boundary must be defined (last part of this lecture)
- Unless we make a further assumption we generally do not know the state of the system in between the initial and final states.

Quasi-equilibrium process assumption

Rate of change limiter...

... the system, moving along a process path from an initial to a final equilibrium state does so slowly enough to ensure the internal equilibrium is maintained.



Why do we need this ?

We can apply the two property rule all the way along the process path.

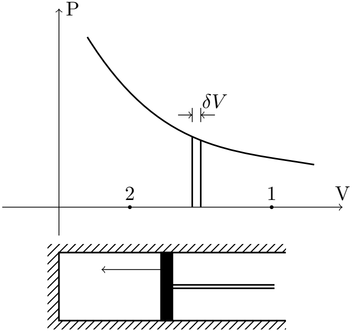
Example

An engine running at 4000 rpm = 76 rps, or one compression cycle takes $\approx$0.015 sec.
The pressure information travels at the speed of sound $\approx 340 ms^{-1}$.
The cylinder stroke is about 0.1 m, so pressure information takes 0.0003 sec to get from the piston to the cylinder head.
This means as the volume changes, the pressure everywhere in the cylinder can be taken as p(t).

Energy Transfer is a Path Property

Need to consider energy change of a fluid..

Work Transfer is *not* a state property (of the fluid), it is a path property (of the process). So we represent the work done between state 1 and 2 as case W_{12}.



- P-V diagrams are very useful for systems involving volume change of the working fluid as a function of pressure: i.e. displacement work: work moving the system boundary.
- Note however if work does follow a quasi-equilibrium path the process should be "slow".
- The term used most is fully resisted.

$\delta V = A\delta x : \delta W = F\delta x = pA\delta x = p\delta V : \int dW = \int pdV.$

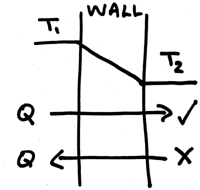
Note :

Remember 1 : State properties are P,V,T,S, work is a process property.
Remember 2 : From the 1st Law ($q_{12} - w_{12} = e_{u,12}$), heat transfer *also* a path property.

Reversible and Irreversible Processes.

A reversible process will be able to....

Reverse its trajectory along a path



characteristics of reversible processes

heat transfer : *no* temperature difference, slow, large areas
motion (work) : zero friction

types of reversible processes

internally reversible : system change is reversible
externally reversible : environment does not change
fully reversible : internal + external

Example

An *irreversible* quasi-equilibrium work process is the slow compression of a gas in an insulated piston-cylinder with friction present.
A *fully reversible* quasi-equilibrium heat transfer process is the very slow expansion of very large and thin walled frictionless piston-cylinder at constant temperature.
An *internally reversible* quasi-equilibrium heat transfer process is the constant volume heating of a cold gas.
This is directly related to entropy generation, which needs to engine efficiency considerations.

Examples of Irreversibility and Entropy Generation

A good engineer is an entropy minimiser

Understanding of the fundamental physics and the design of technology that minimises irreversible entropy generation. !

Examples of irreversible entropy generation in thermofluids

- viscous friction (Shear Stresses) in fluid flow - leads to pressure drops in pipes, drag on objects.
- Turbulent flow - much larger pressure drops in pipes, drag on objects.
- Heat transfer across a temperature difference.
- Unrestrained expansion.
- Shock waves in compressible fluids (not covered here).
- Mixing of different fluids (not covered here).
- Phase change (not covered here).
- Chemical Reaction/Combustion (not covered here).

Also attributed to a wide range of more philosophical processes

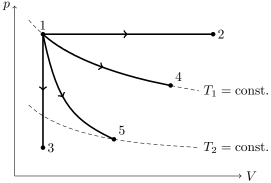
- Information Quality (signal processing, code breaking etc, not covered here!).
- Heat Death of the Universe (not covered here!!).
- Evolution of Life (not covered here!!!).

Process Types (1/4) : Isochoric (Constant Volume) heat transfer.

Used to model *fast* Combustion : Petrol Engines

Typically the gas is heated, increasing the pressure, in a rigid container.

Constant Volume (no work) $V_1 = V_3$, $\frac{p}{T} = const$



- Initial, final states are related by : $\frac{p_1 V_1}{T_1} = \frac{p_3 V_3}{T_3}$.
- Conservation of Energy : $Q_{13} - W_{13} = E_{u,13}$ or $q_{13} - w_{13} = e_{u,13}$
- Work transfer : $W_{13} = 0$
- Heat transfer : $Q_{13} = W_{13} + E_{u,13} = mC_v(T_3 - T_1)$
- or.. $q_{13} = w_{13} + e_{u,12} = C_v(T_3 - T_1)$
- $Q_{13} = E_{u,13}$ (heat transfer is the internal energy change)

Reversibility

The heat transfer occurs over the temperature range: the process is not reversible.

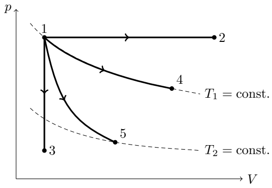
Process Types (2/4) : Isobaric (Constant Pressure) Expansion/Compression.

Slow/steady combustion : boilers, gas turbines

Energy (heat) transfer) raises the internal energy and does displacement work. The temperature rises to maintain the pressure as volume increases.

Constant Pressure, $p_1 = p_2$, $\frac{V}{T} = const$

- Initial, final states are related by : $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$.
- Conservation of Energy : $Q_{12} - W_{12} = E_{u,12}$ or $q_{12} - w_{12} = e_{u,12}$
- Work transfer : $W_{12} = \int dW = \int pdV = p(V_2 - V_1)$. or $w_{12} = \int dw = \int pdv = p(v_2 - v_1)$
- Heat transfer : $Q_{12} = W_{12} + E_{u,12} = mC_p(T_2 - T_1) = p(V_2 - V_1) + mC_v(T_2 - T_1)$
- or.. $q_{12} = w_{12} + e_{u,12} = C_p(T_2 - T_1) = p(v_2 - v_1) + C_v(T_2 - T_1)$
- $Q_{12} = E_{h,12} = E_{u,12} + p\Delta V$ (heat transfer is the enthalpy change)



Reversibility

The heat transfer occurs over the temperature range: the process is not reversible. (friction makes it worse).

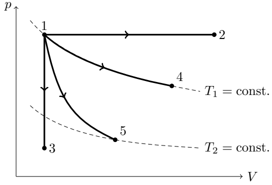
Process Types (3/4) : Isothermal (Constant Temperature) Expansion/Compression.

Impractically slow expansion : theoretical engine efficiency benchmarking

Heat transfer must occur over a negligible temperature difference - large areas..

Constant Temperature, $T_1 = T_4$ and thus $pV = const$

- Initial, final states are related by : $\frac{p_1V_1}{T_1} = \frac{p_4V_4}{T_4}$.
- Conservation of Energy : $Q_{14} - W_{14} = E_{u,14}$ or $q_{14} - w_{14} = e_{u,14}$
- Work transfer :$W_{14} = \int dW = \int pdV = p_1V_1 \int \frac{dV}{V} = p_1V_1 ln\left(\frac{V_4}{V_1}\right)$
- Heat transfer : $Q_{14} = W_{14}$. Note that $E_{u,14} = mC_v(T_4 - T_1) = 0$
- or.. $q_{14} = w_{14}$ (heat transfer is the work done)



Reversibility

All heat transfer occurs at the isotherm temperature: the process is reversible if frictionless.

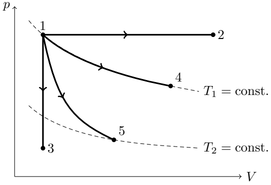
Process Types (4/4): Adiabatic (zero heat transfer) Expansion/Compression.

Expansion occurs in an insulated container

A good approximation for the power and compression strokes in internal combustion engines.

p-v relationship $pv^\gamma = constant, \gamma = C_p/C_v$

- Initial, final states are related by : $\frac{p_1V_1}{T_1} = \frac{p_5V_5}{T_5}$.
- Conservation of Energy : $Q_{15} - W_{15} = E_{u,15}$ or $q_{15} - w_{15} = e_{u,15}$
- Work transfer :$W_{15} = \int dW = \int pdV = p_1V_1^\gamma \int \frac{dV}{V^\gamma}$
- Heat transfer : $Q_{15} = 0$
- Conservation of Energy : $-W_{15} = E_{u,15} = mC_v(T_5 - T_1)$



Reversibility

Zero heat transfer: the process is reversible if frictionless.

Summary and Suggested Reading

Lecture Summary

- Equilibrium and the Zeroth Law of thermodynamics.
- States, Processes, paths, Initial/final States.
- Quasi-equilibrium assumption for processes.
- Reversible and irreversible processes.
- Heat and work as Process properties.
- P-V diagrams.
- Types of Processes.

Suggested Further Reading

- Cengel and Boles Section 1.6 (internal equilibrium), Section 1.7 (process/path QE process), Section 1.8 (zeroth law), Section 6.6 (reversible/irreversible process).
- Shrimpton, Section 2.9.1-2.9.8.

Suggested Further Study

- Try chapter 5 questions, worked solution 5.7

Lecture 7: Change in thermofluid systems (2/2)

Last Lecture we covered:

- Equilibrium and the Zeroth Law of thermodynamics.
- States, Processes, paths, Initial/final States.
- Quasi-equilibrium assumption for processes.
- Reversible and irreversible processes.
- Heat and work as Process properties.
- P-V diagrams.
- Types of Processes.

This lecture we are going to cover:

- Macroscopic Entropy as a state variable
- Reversibility again.
- T-S diagrams of key processes.
- TdS equations.
- Irreversible change and entropy rise.

State of knowledge thus far.

A reminder..

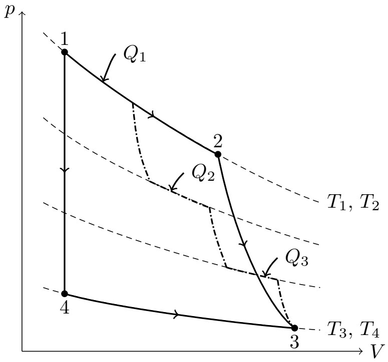
- We have defined three state variables: T (or e_u), p and V (or ρ). Any two of these define the state of a given mass of gas via the two property rule.
- Process variables include the work and energy transfers across the system boundary.
- These, together with an initial and final state, define the path of the process.
- Enthalpy is another variable, that represents the internal energy and the displacement work summed.
- These are the results of different types of reversible/irreversible process operations, we consider 4.
- isothermal and adiabatic compression/expansion, and heat transfers at constant pressure and constant volume.
- We also introduced a variable called entropy, and defined it from a *microscopic* viewpoint in terms of the number of available states in either velocity or position.

We now discuss *macrosopic* entropy...

Two Processes : Isothermal Expansion, Followed by Adiabatic expansion.

We have two points defined on a P-V diagram, 1 (initial state) and 3 (final state)

We analyze various paths and the energy budget along them



- First let us look at the process path $1 \rightarrow 2 \rightarrow 3$.
- This is a reversible isothermal expansion followed by a reversible adiabatic expansion.
- $1 \rightarrow 2$: Isothermal expansion. Note also that the heat provided to the gas is all provided at a single temperature, $T_1 = T_2$ and thus the internal energy does not change. $Q_{12} = W_{12}$.
- $2 \rightarrow 3$: Adiabatic expansion: $E_{u,23} = -W_{23}$.

Summary

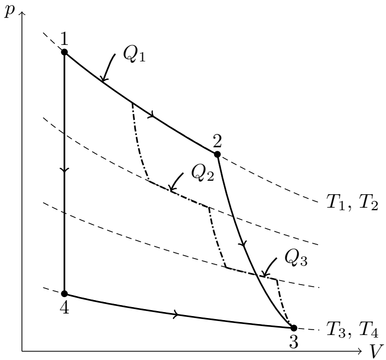
During the process $1 \rightarrow 2 \rightarrow 3$: Q_{12} has been supplied to the gas and the internal energy has changed.

Two Processes: Adiabatic Expansion, Followed by Isothermal expansion.

We have two points defined on a P-V diagram, 1 (initial state) and 3 (final state)

We analyze various paths and the energy budget along them

- Now let us look at the process path $1 \rightarrow 4 \rightarrow 3$.
- This is a reversible adiabatic expansion followed by a reversible isothermal expansion.
- $1 \rightarrow 4$: Adiabatic expansion: $E_{u,14} = -W_{14}$.
- $4 \rightarrow 3$: Isothermal expansion. Note also that the heat provided to the gas is all provided at a single temperature, $T_4 = T_3$ and thus the internal energy does not change. $Q_{43} = W_{43}$



Summary

During the process $1 \rightarrow 4 \rightarrow 3$ Q_{43} has been supplied to the gas and the internal energy has changed.

Differences in the process routes $1 \rightarrow 2 \rightarrow 3$ and $1 \rightarrow 4 \rightarrow 3$.

Similarities

- Paths $1 \rightarrow 2 \rightarrow 3$ and $1 \rightarrow 4 \rightarrow 3$ are both reversible.
- same start and end at the same points.
- same internal energy change.

On paths $1 \rightarrow 2 \rightarrow 3$ and $1 \rightarrow 4 \rightarrow 3$:

Differences

- The heat transfers are different.
- The work transfers are different.
- Because the initial and final states are the same irrespective of the path, then the entropy (a state variable) must also change by a defined amount.

We need a way to describe the entropy change independent of the path.....in terms of process variables, ie heat and work transfers !

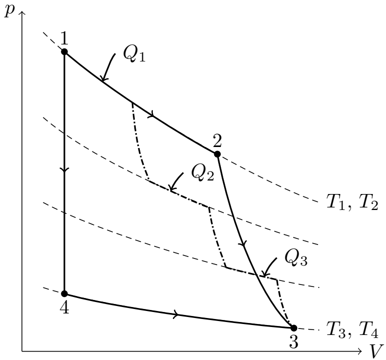
Macroscopic Entropy definition.

The quantity of heat added divided by the temperature at which it is added along the path is independent of the path here.

$\frac{Q_{43}}{T_3} = \frac{Q_{12}}{T_1} = S_{43} = S_{12} = S_{13}$

We could choose any number of steps..

$S_{13} = \sum_{n=1}^{N} \frac{Q_n}{T_n} = \int_1^3 \frac{dQ}{T}$



This gives us our entropy definition

- Because the variable is independent of the path, it must be a state property.
- Thus there are 4 state properties, S,P,V,T that define a gas.
- If this is true we must be able to define relationships between them because of the two property rule.

Entropy Difference for an ideal gas: TdS equations (1/2)

Analysing the entropy change to a gas in a volume under constant pressure heating

- We transfer a *small* amount of energy δq and allow the volume to change to maintain constant pressure.
- From our definitions : $\delta q = C_p \delta T$ and $\delta q = C_V \delta T + p\delta v$
- Dividing by T, $\frac{\delta q}{T} = C_V \frac{\delta T}{T} + \frac{p}{T}\delta v$
- Making use of the gas law : $\frac{\delta q}{T} = C_V \frac{\delta T}{T} + R\frac{\delta v}{v}$
- Integrating: $\int \frac{dq}{T} = C_V \int \frac{dT}{T} + R\int \frac{dv}{v}$ or...
- $s_{12} = C_v \int_1^2 \frac{dT}{T} + R\int_1^2 \frac{dv}{v}$.

This is known as the first TdS equation it is:
$s_{12} = C_V ln\frac{T_2}{T_1} + Rln\frac{v_2}{v_1}$.
This is the *intensive* form - entropy here has units of J/kgK.
It gives us the entropy change in terms of the change in two state variables T and v.
The *extensive* form is $S_{12} = mC_V ln\frac{T_2}{T_1} + mRln\frac{V_2}{V_1}$.

Entropy Difference for an ideal gas: TdS equations (2/2)

For the 2nd TdS equation we use the enthalpy definition

Starting from $s_{12} = C_V ln\frac{T_2}{T_1} + Rln\frac{V_2}{V_1}$, we can employ the enthalpy relation $e_h = e_u + pv$.
Where...$de_h = de_u + d(pv) = de_u + pdv + vdp$.
Rearranging this equation $de_h - vdp = de_u + pdv$.
We note that since $de_h = C_p dT$ and $de_u = C_v dT$. Dividing by T..
Rearranging this equation $Cp\frac{dT}{T} - \frac{v}{T}dp = C_v\frac{dT}{T} + \frac{p}{T}dv$.
Making use of the gas law : $Cp\frac{dT}{T} - \frac{R}{p}dp = C_v\frac{dT}{T} + \frac{R}{v}dv$.
The RHS is TdS from the first TdS equation.
So the 2^{nd} TdS equation is: $s_{12} = C_V ln\frac{T_2}{T_1} - Rln\frac{p_2}{p_1}$.

Summary

- First TdS Equation: $s_{12} = C_V ln\frac{T_2}{T_1} + Rln\frac{V_2}{V_1}$ or $Tds = de_u + pdv$.
- Second TdS Equation: $s_{12} = C_p ln\frac{T_2}{T_1} - Rln\frac{p_2}{p_1}$ or $Tds = de_h - vdp$.

Key Points

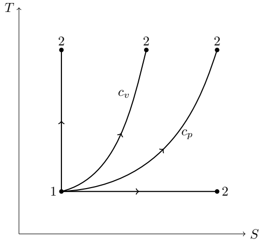
- We used the entropy definition and the 1^{st} Law and the enthalpy definition to derive the TdS equations
- They apply to both reversible and irreversible processes, and not just the reversible example given.

Process Types (1/4) : Isochoric (Constant Volume) heat transfer.

Used to model *fast* Combustion : Petrol Engines

Typically the gas is heated, increasing the pressure, in a rigid container.

Constant Volume (no work) $V_1 = V_2$, $\frac{p}{T} = const$



- Initial, final states are related by : $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$.
- Conservation of Energy : $Q_{12} - W_{12} = E_{u,12}$ or $q_{12} - w_{12} = e_{u,12}$
- $W_{12} = 0, Q_{12} = mC_V(T_2 - T_1)$.
- First TdS Equation: $S_{12} = mC_V ln \frac{T_2}{T_1} + mR ln \frac{V_2}{V_1}$
- 1^{st} TdS Equation: $S_{12} = mC_V ln \frac{T_2}{T_1}$.

Reversibility

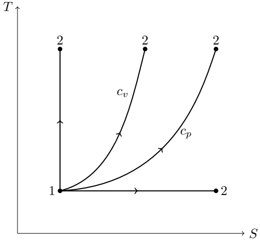
The heat transfer occurs over the temperature range: the process is not reversible.

Process Types (2/4) : Isobaric (Constant Pressure) Expansion/Compression.

Slow/steady combustion : boilers, gas turbines

Energy (heat) transfer) raises the internal energy and does displacement work. The temperature rises to maintain the pressure as volume increases.

Constant Pressure, $p_1 = p_2$, $\frac{V}{T} = const$



- Initial, final states are related by : $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$.
- Conservation of Energy : $Q_{12} - W_{12} = E_{u,12}$ or $q_{12} - w_{12} = e_{u,12}$
- Work transfer : $W_{12} = \int dW = \int pdV = p(V_2 - V_1)$.
- Heat transfer : $Q_{12} = W_{12} + E_{u,12} = mC_p(T_2 - T_1) = p(V_2 - V_1) + mC_v(T_2 - T_1)$
- Second TdS Equation: $S_{12} = mC_p ln\frac{T_2}{T_1} - mRln\frac{p_2}{p_1}$
- 2^{nd} TdS Equation: $S_{12} = mC_p ln\frac{T_2}{T_1}$.

Reversibility

The heat transfer occurs over the temperature range: the process is not reversible. (friction makes it worse).

Process Path Types (3+4 of 4): Adiabatic + Isothermal volume increase/decrease.

Isothermal Change

Change is at constant temperature.

$W_{12} = p_1 V_1 ln\left(\frac{V_2}{V_1}\right)$

$Q_{12} = W_{12}$ because of zero change in internal energy.

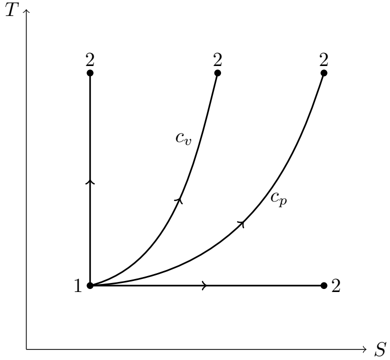
$S_{12} = mRln\frac{V_2}{V_1} = \frac{Q_{12}}{T_1}$.

All heat transfer occurs at the isotherm temperature: the process is reversible if frictionless.

Adiabatic Change

zero heat transfer during the change.

$Q_{12} = 0; S_{12} = 0$, if zero friction.



Process Terminology definitions.

4 Types of Change :

For processes where one of the 4 state variables P,V,T,S stays constant the following terminology is appropriate.

- Isobaric - constant pressure.
- Isochoric - constant volume.
- Isothermal - constant temperature.
- Isentropic - constant entropy.

Important Points to Remember

- Of these isobaric and isochoric process cannot be reversible because the heat transfer between system and environment occurs over a temperature difference.
- They are not isentropic because of the heat transfer and because of the irreversibility.
- The isothermal process is reversible because the heat transfer occurs over a negligible temperature difference.
- However because of the heat transfer the process cannot be isentropic.
- The adiabatic process is isentropic if it is reversible.
- Any other losses (friction for instance) are an irreversible change, they create entropy.

Using P-V and T-S diagrams.

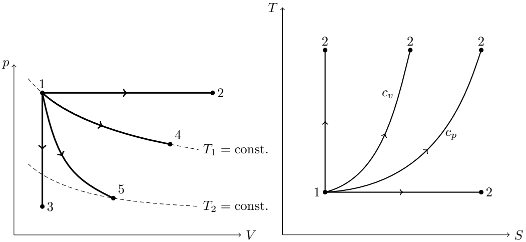
For processes integrals in P-V and T-S diagrams have direct physical interpretation.

$W_{12} = \int dW = \int pdV$.

- $W > 0$ for $dV > 0$ ($p > 0$ always).

$Q_{12} = \int dQ = \int TdS$.

- Note $Q > 0$ for $dS > 0$ ($T > 0$ always).



Sign Convention during a single process

- Changes in P,V,T,S are due to Q and W transfers between our system and the environment defined by $Q_{12} - W_{12} = E_{u,12}$. $Q > 0$ when adding heat to the system. $W > 0$ when system does the work.
- Changes in P,V,T,S for any single process of the system may be > 0 or < 0 because W,Q change likewise.
- When considering cycles, change in $S_{total} \geq 0$ only is possible (we have to consider the environment entropy change as well !).

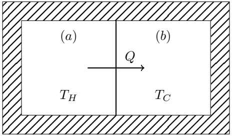
Example 1 : Entropy as an energy quality measure : Heat Flow between two chambers.

We have an insulated rigid chamber with a partition separating two equal sub-volumes.

- At time 1 the fluid in one volume is hot, at temperature T_H,the other cold at a temperature T_C.
- At time 2 both sub-volumes have the same temperature due to heat transfer through the partition.

From time 1 to 2

- No work transfer. No heat transfer.
- Therefore from the first law the internal energy in the chamber has not changed.
- A heat transfer has occurred between sub-volumes and if we consider the each sub-volume a separate system (a and b) then the initial reversible entropy change in each system is.
- $S_{a,12} = -\frac{Q}{T_H}, S_{b,12} = \frac{Q}{T_C},$
  $S_{12} = S_{a,12} + S_{b,12} = -\frac{Q}{T_H} + \frac{Q}{T_C} > 0.$



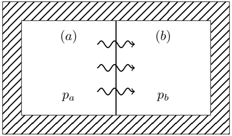
Summary

Zero Energy Change, but Entropy has increased. Our 'extractable' energy is less.

Example 2 : Entropy as an energy quality measure : Pressure leakage between two chambers.

we have the same rigid container and this time the initial state 1 is the pressure in the two sub-volumes is different ($p_a = 2p_b$ say)

- No work transfer. No heat transfer.
- Internal energy in the chamber has not changed.
- The entropy change in each system is characterized by the pressure change.
- $p_a V = m_a RT, p_b V = m_b RT, m_a = 2m_b, p_2 2V = (m_a + m_b)RT$.
- $m_a = 2m_b, p_2 = \frac{3m_a RT}{4V}, \frac{p_2}{p_a} = \frac{3}{4}$: Final pressure goes down w.r.t. p_a.
- $m_b = \frac{m_a}{2}, p_2 = \frac{3m_a RT}{2V}, \frac{p_2}{p_b} = \frac{3}{2}$: Final pressure goes up w.r.t. p_b.
- $S_{a,12} = -2m_b R ln\left(\frac{p_2}{p_a}\right), S_{b,12} = -m_b R ln\left(\frac{p_2}{p_b}\right)$.
- $S_{12} = S_{a,12} + S_{b,12} = -Rm_b\left[2ln\left(\frac{p_2}{p_a}\right) + ln\left(\frac{p_2}{p_b}\right)\right] > 0$ (check it!).

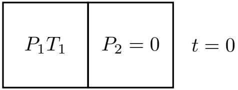


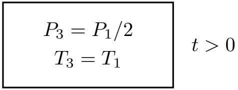
Summary

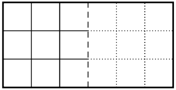
Entropy increases, energy quality decreased, lost useful energy forever.

Example 3 : The molecular speed distribution of our molecules in our box as a function of V.

Consider molecules on one half of box, where the other half is a empty vacuum







- The partition is removed and the molecules over time fill the box.
- They are very very unlikely to ever go back - the process is irreversible.
- The gas has expanded against a vacuum so no work has been done.
- The box is insulated, there is no heat transfer and the molecules have lost no internal energy.
- The macroscopic quality of the energy has however has degraded because of the loss of pressure.

In terms of microscopic "range of possible states" each molecule has more position states when the partition is removed. If we divide each section into 9 areas, at $t = 0$ the probability of finding a molecule in any one area is 1 in 9, after it is 1 in 18. This increases the molecular entropy $S = k_B lnN_S$.

Relating the microscopic and the macroscopic entropy definitions

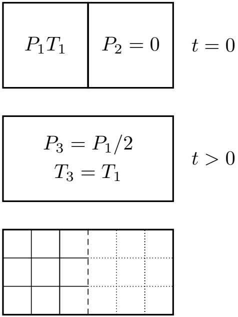
Macroscopic Entropy Change - 1st TdS equation

- $S_{12} = mRln\frac{V_2}{V_1} = mRln2$.
- Recall $k_B = \frac{R_u}{N_A}$ and $m_u = \frac{m}{n}$ so:
- $S_{12} = mRln2 = \frac{N}{N_A}R_u ln2 = Nk_B ln2 > 0$

The molecular entropy definition was $S = k_B ln N_S$

- $S_1 = k_B ln9$, $S_2 = k_B ln18$
- $S_{12} = Nk_B ln2 > 0$

Each of the N molecules have picked up $k_B ln2$ of entropy by increasing the number of possible states by $ln2$.



Summary and Suggested Reading

Lecture Summary

- Macroscopic Entropy as a state variable.
- T-S diagrams of key processes.
- TdS equations.
- Irreversible change and entropy rise.
- A link between microscopic and macroscopic entropy.

Suggested Further Reading

- Cengel and Boles - Chapter 7 (entropy)
- Cengel and Boles - Section 7.4 (T-S diagams)
- Cengel and Boles - Section 7.7 (TdS equations)
- Cengel and Boles - Section 6.6 (irreversible change)
- Shrimpton, Section 2.9.9 - 2.9.13

Suggested Further Study

- Can complete chapter 5.

Lecture 8: Cycles and Heat Engines

Last Lecture we covered:

- Macroscopic Entropy as a state variable.
- T-S diagrams of key processes.
- TdS equations.
- Irreversible change and entropy rise.
- A link between microscopic and macroscopic entropy.

This lecture we are going to cover:

- Cycles : a sequence of processes.
- Heat Engines
- Thermal Reservoirs and heat rejection
- Efficiency of heat engines.

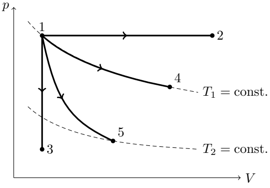
Reminder: Isochoric (Constant Volume) pressure increase/decrease

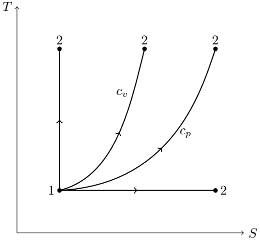
Used to model *fast* Combustion : Petrol Engines

Typically the gas is heated, increasing the pressure, in a rigid container.

Constant Volume (no work) $V_1 = V_3$, $\frac{p}{T} = const$

- Initial, final states are related by : $\frac{p_1 V_1}{T_1} = \frac{p_2 V_3}{T_3}$.
- Conservation of Energy : $Q_{13} - W_{13} = E_{u,13}$ or $q_{13} - w_{13} = e_{u,13}$
- $W_{13} = 0, Q_{13} = mC_V(T_3 - T_1)$.
- First TdS Equation: $S_{13} = mC_V ln\frac{T_3}{T_1} + mRln\frac{V_3}{V_1}$
- 1^{st} TdS Equation: $S_{13} = mC_V ln\frac{T_3}{T_1}$.





Reversibility

The heat transfer occurs over the temperature range: the process is not reversible.

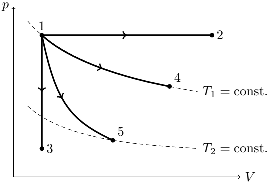
Reminder: Isochoric (Constant Pressure) pressure increase/decrease

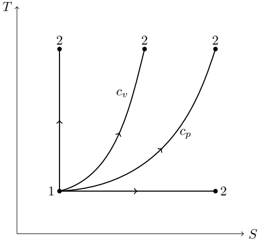
Slow/steady combustion : boilers, gas turbines

Energy (heat) transfer) raises the internal energy and does displacement work. The temperature rises to maintain the pressure as volume increases.

Constant Pressure, $p_1 = p_2$, $\frac{V}{T} = const$

- Initial, final states are related by : $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$.
- Conservation of Energy : $Q_{12} - W_{12} = E_{u,12}$ or $q_{12} - w_{12} = e_{u,12}$
- Work transfer : $W_{12} = \int dW = \int pdV = p(V_2 - V_1)$.
- Heat transfer : $Q_{12} = W_{12} + E_{u,12} = mC_p(T_2 - T_1) = p(V_2 - V_1) + mC_v(T_2 - T_1)$
- Second TdS Equation: $S_{12} = mC_p ln\frac{T_2}{T_1} - mRln\frac{p_2}{p_1}$
- 2^{nd} TdS Equation: $S_{12} = mC_P ln\frac{T_2}{T_1}$.





Reversibility

The heat transfer occurs over the temperature range: the process is not reversible. (friction makes it worse).

Reminder : Adiabatic+Isothermal volume increase/decrease

Isothermal Change

Change is at constant temperature.

$W_{12} = p_1 V_1 ln\left(\frac{V_2}{V_1}\right)$

$Q_{12} = W_{12}$ because of zero change in internal energy.

$S_{12} = mRln\frac{V_2}{V_1} = \frac{Q_{12}}{T_1}$.

All heat transfer occurs at the isotherm temperature: the process is reversible if frictionless.

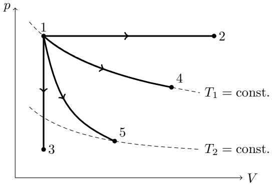
Adiabatic Change

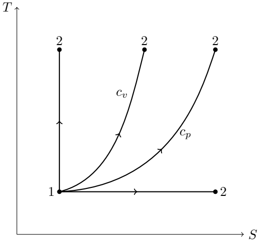
$pv^{\gamma} = Const.$

zero heat transfer during the change.

$Q_{12} = 0; S_{12} = 0$, if zero friction.

$-W_{12} = E_{u,12}$

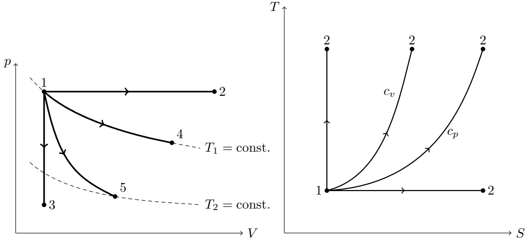




Equations governing single closed processes

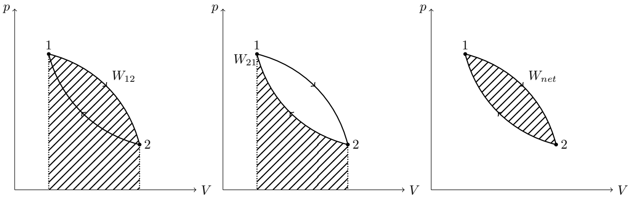
Key Equations :

- Conservation of Energy: $Q_{12} - W_{12} = E_{12}$
- Gas Law: $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$
- Entropy definition : $S_{12} = mC_v \ln \frac{T_2}{T_1} + mR \ln \frac{V_2}{V_1}$, $S_{12} = mC_p \ln \frac{T_2}{T_1} - mR \ln \frac{P_2}{P_1}$
- $W_{12} = \int dW = \int pdV$: Note $W > 0$ for $dV > 0$ ($P > 0$ always)
- $Q_{34} = \int dQ = \int TdS$: Note $Q > 0$ for $dS > 0$ ($T > 0$ always)



A cycle of processes

Consider a two process cycle - start,end at same point.



In the simple cycle shown above..

- W_{12} is + ve (V increases) and W_{21} is -ve. The *net* work is the area *enclosed* by the cycle.
- The net work is clearly > 0 and for clockwise cycles this is always the case.
- The *same* is true on a T-S diagram, where the net heat transfer is defined by the area enclosed on that cycle.

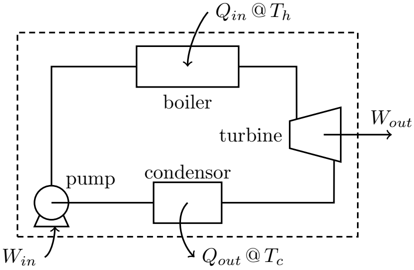
Important Point

- Initial + final points on the cycle are the *same*, the cycle internal energy is zero.
- Leading to the first law for cycles to be $\sum Q - \sum W = 0$.
- Since $W_{net} = Q_{net}$, area on T-s and P-v diagrams show the same thing.

Heat engines (things that produce work from heat)

A heat engine has the following characteristics :

- They access heat energy from a high temperature source.
- They produce/use some work
- They operate on a cycle
- They access heat energy to a low temperature source.



Examples are:

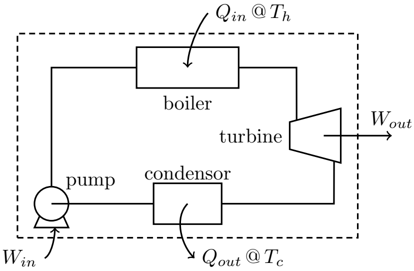
- Internal combustion engines
- Steam/Gas turbines
- Refrigerators
- Heat Pumps

Up till now we have only examined single processes, and not concerned ourselves with the impact of dumping heat to the environment.
As we shall see, it is crucial for good efficiency.

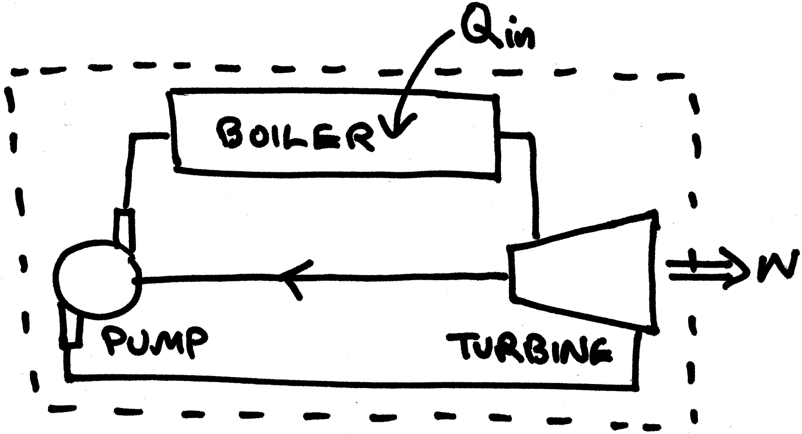
2^{nd} Law of thermodynamics : Kelvin-Planck Statement

Heat engines run cycles that seem to require us to reject waste energy to a heat sink.

- This leads to several consequences enshrined in one of the more useful interpretations of the 2^{nd} Law.
- The Kelvin-Planck statement.
- *It is impossible for any heat engine to receive heat from a single thermal reservoir and produce an equivalent amount of work.*
-
- *A cyclically operating engine must reject heat to a low temperature 'sink' as well as receive heat from a high temperature source*
- Which means no heat engine can be 100% efficient



Example : 2^{nd} Law Violation

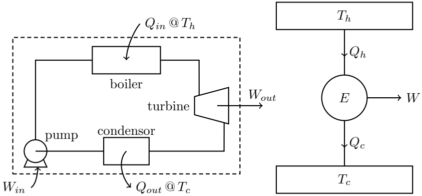


In the above sketch, 2^{nd} Law is violated because there is no low temperature sink for this engine cycle.

Heat Engines and Thermal Reservoirs

To calculate efficiency, we only need the energy transfers and the work known

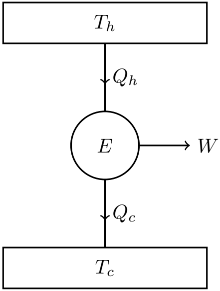
- "E"is the entire heat engine, the system boundary is the circle around the E
- Th and Tc are the hot and cold thermal reservoir (a very large thermal mass ensuring that whatever heat is added/taken from it the temperature stays the same
- Q_h and Q_c are the heat transfers to/from the hot and cold reservoirs respectively.
- W is the net work crossing the system boundary.



A key simplification over the 1st Law is that the signs on Q and W are ignored here, the direction of heat and work flow is defined by arrows on the diagram.

Heat Engine Efficiency

Efficiency is what you get in terms of what you pay for



- In terms of heat engines :'*what you get* is the work output, W
- *what you pay for* is the energy extracted from the high temperature reservoir.
- Note that the energy donated to the cold reservoir is not counted, it is wasted energy
- Heat Engine Thermal Efficiency: $\eta_{th} = \frac{W}{Q_h}$
- For a cycle, $\sum W = \sum Q$ and therefore $W = Q_h - Q_c$
- Therefore $\eta_{th} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$

Notes

The larger Q_h can be and the smaller Q_c can be the more efficient our heat engine will be.
This is a theoretical limit-practical inefficiencies, eg friction add to this.

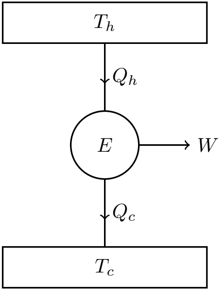
Heat Engine Energy Quality

The heat engine efficiency of any engine is defined..

$\eta_{th} = 1 - \frac{Q_c}{Q_h}$.

The efficiency of a reversible engine is defined..

by ONLY on the hot and cold reservoir temperatures.
It does not depend on the type of processes, the working fluid or anything else.
the important relation is,
$\eta_{th,rev} = 1 - \frac{T_c}{T_h}$



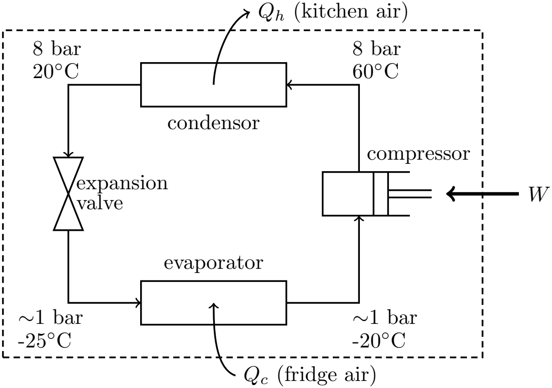
Example

This *instantly* gives an idea of energy quality. If the hot and cold T are 1200K and 300K
$\eta_{th,rev} = 0.75$, reduce the hot T to $600K$ and $\eta_{th,rev} = 0.50$.
Clearly some energy sources are more efficiently used than others.

Carnot Efficiency

The reversible heat engine efficiency is sometimes known as the Carnot Efficiency.
It is the efficiency that all engines aspire to !

Example of a Reversed Heat Engine



Note this is an example only : we do not cover phase change systems in this course.

Reversed Heat Engine Efficiency

We can't call it *efficiency*

Reversed heat engines use work to move heat from a cold to a hot reservoir.

Remember efficiency is *what you get* in terms of *what you pay for*.

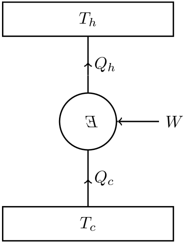
"What you pay for" is obvious : the electrical work to run the engine.

What we get *depends* on the desired outcome of the engine.

ALSO :"efficiencies" here can be > 1, therefore we call this the called Coefficient of Performance

Heat Pumps : The useful energy is the hot heat flow (to a house) : $COP_h = \frac{Q_h}{W}$

Refridgerators : The useful energy is the cold heat flow : $COP_r = \frac{Q_c}{W}$



Summary of this Lecture/Suggested Reading

Lecture Summary

- Cycles : a sequence of processes.
- Description of a spark ignition engine cycle and the model system analogue.
- Heat Engines
- Thermal Reservoirs and heat rejection
- Efficiency of heat engines.

Suggested Further Reading

- Cengel and Boles : chapter 9
- Shrimpton section $3.1 - 3.7$

Suggested Further Study

- Try chapter 6 questions, worked solution 6.23

Lecture 9: The Carnot Cycle: The Impossible Engine

Last Lecture we covered:

- Cycles : a sequence of processes.
- Description of a spark ignition engine cycle and the model system analogue.
- Heat Engines
- Thermal Reservoirs and heat rejection
- Efficiency of heat engines.

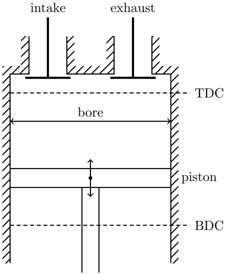
This lecture we are going to cover:

- Introduction of the Carnot cycle
- Why it is the most efficient cycle possible and a comparator to real engine cycles.
- Demonstration of energy quality and comparison of Carnot efficiency to real engines.

Building an impossible engine...

The Carnot Engine is...

- Fully reversible
- Consists of 4 processes.
- It has a frictionless piston-cylinder arrangement where the cylinder walls are perfectly insulated.
- The cylinder head has a removable perfect insulation.
- It may be replaced by a heat source a heat sink.



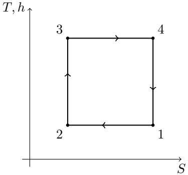
It is not a practical engine

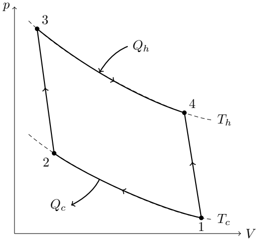
- A theoretical engine - the most efficient any heat engine can possibly be.
- Understanding why a Carnot heat engine is efficient is to understand how practical heat engines have losses.
- And then add *real* losses that are not the mechanical practicalities of friction, leaks etc.

Processes of a Carnot Engine.

4 processes..

- Process 12: Reversible isothermal compression : Heat Sink in place on cylinder head.
- Process 23: Reversible adiabatic compression : insulated cylinder head.
- Process 34: Reversible isothermal expansion : Heat Source in place on Cylinder head
- Process 41: Reversible adiabatic expansion : insulated cylinder head.





Fully Reversible Engine but..

- The impossibility relates to the isothermal processes - (reversible heat transfer).
- This means the heat transfer takes an impossibly long time over an impossibly large area.

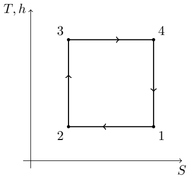
The Carnot cycle as a maximum efficiency engine.

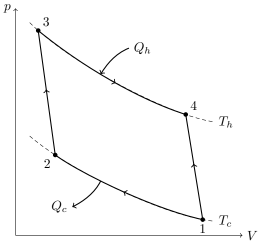
All the heat transfer occurs at the heat sink/source temperature

- The Carnot Efficiency is therefore (as noted above)
- $\eta_{th} = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h}$
- $\eta_{th} = 1 - \frac{T_c}{T_h}$

For all practical model systems

- Heat addition/rejection steps occur over a temperature range.

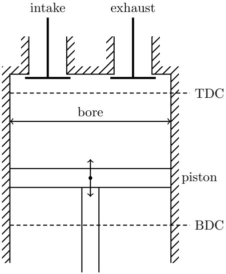




Reciprocating Engines

Overview and Terminology

- Piston moves a certain stroke between Bottom and Top Dead Centre (BDC and TDC)
- Clearance volume @TDC = V_{min}
- Total volume @BDC =V_{max}
- Volume compression ratio $r = V_{max}/V_{min}$



Variations

- SI engines use smaller compression ratios, lighter fuels, inject fuel various ways
- CI engines use larger compression ratios, inject heavier fuels directly into the cylinder to auto-ignite the combustion mixture.
- Vast scale/speed range, F1 engines 15000rpm, 30cc model aircraft engines, marine Diesels the size of houses.

Net Work Measurement

The area enclosed by a cycle on a PV diagram defines the net work done by that cycle.

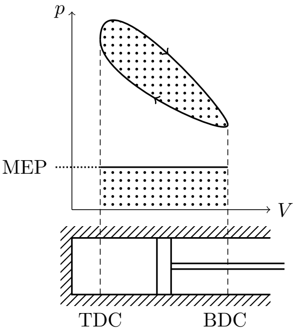
A common term to characterise engine performance is the mean effective pressure (MEP).

It is a ficticous pressure which if acted on the piston during the entire piston motion from BDC to TDC would produce the same net work obtained by the cycle. Ie the two shaded areas are equal.

$$W = MEP.Area.Stroke = MEP.(sweptvol)$$

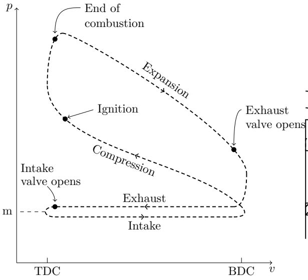
Or

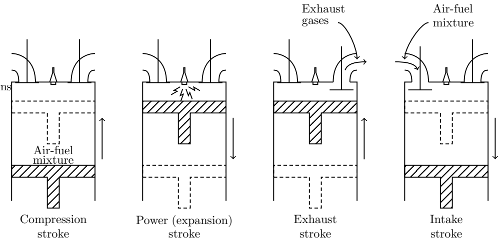
$$MEP = \frac{W}{V_{max} - V_{min}}$$



Practical Heat Engine : 4 Stroke Spark Ignition IC Engine

- Compression stroke : from BDC to TDC (note clearance volume), both valves closed (usually fuel injected during the intake stroke)
- Power/expansion stroke : Injection of liquid fuel, ignition of compressed mixture (near constant volume process with pressure/temperature rise), expansion of combustion mixture
- Exhaust stroke : Exhaust valve opens, hot gases pushed out of cylinder by stroke
- Intake stroke : exhaust valve closes, intake opens, fresh cooler air charge sucked into cylinder

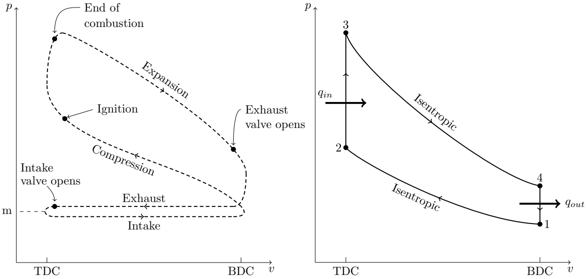




The Otto Cycle : A model of the 4 Stroke SI IC Engine

Here the working fluid stays in the cylinder

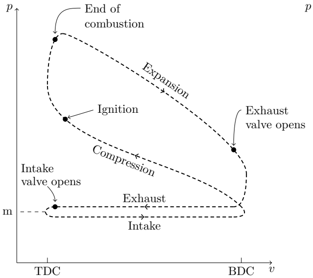
- Process 1-2 : Isentropic/Adiabatic compression
- Process 2-3 : constant volume heat addition (modelling the fuel injection and combustion, +ve)
- Process 3-4 : Isentropic expansion (power stroke)
- Process 4-1 : constant volume heat rejection (-ve)

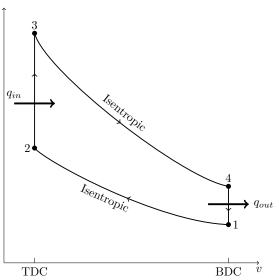


Air Standard Assumptions for using a system basis for an IC Engine

The assumptions used to model an IC engine are

- The working fluid is air which continuously circulates in a closed and always behaves as an ideal gas
- All the processes are internally reversible.
- The combustion process is a heat addition process from an external source
- The exhaust process is a heat rejection process that restores the working fluid back to its original state



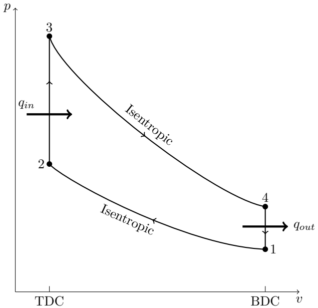


Net Work for an Otto engine cycle

Example

Calculate the *net* work from a cycle

- $NetWork = W_{12} + W_{32}, W_{23} = W_{41} = 0$
- Isentropic/adiabatic process: $pV^{\gamma} = K$
- $W_{12} = \int_1^2 p\mathrm{d}V = K\int_1^2 V^{-\gamma}\mathrm{d}V = \frac{K}{1-\gamma}(V_2^{1-\gamma} - V_1^{1-\gamma})$ $= \frac{1}{1-\gamma}(P_2V_2^{\gamma}V_2^{1-\gamma} - P_1V_1^{\gamma}V_1^{1-\gamma}) = \frac{(P_2V_2 - P_1V_1)}{1-\gamma}$
- $W_{34} = \frac{(P_4V_4 - P_3V_3)}{1-\gamma}$



Note : W_{34} is +ve and W_{12} is -ve

$NetWork = \frac{(P_4V_4 + P_2V_2 - P_3V_3 - P_1V_1)}{1-\gamma}$

Or the area inside the PV diagram – the larger the area (better the shape) the more work is obtained.

Hint : Often easier to work out the net heat transfer !

Summary of this Lecture/Suggested Reading

Lecture Summary

- Introduced the Carnot Cycle, what is required to in theory build one
- Outlined why it is the cycle with the highest possible efficiency – lossless and energy transfers occur at the reservoir temperatures
- Provides a baseline with which to bench-mark the performance of real heat engines

Suggested Further Reading

- Cengel and Boles : chapter 9
- Shrimpton section 3.8

Suggested Further Study

- Complete chapter 6 questions.

Internal Combustion Engines : Practical Engine Cycles

Last Lecture we covered:

- Introduced the Carnot Cycle, what is required to in theory build one
- Outlined why it is the cycle with the highest possible efficiency – lossless and energy transfers occur at the reservoir temperatures
- Provides a baseline with which to bench-mark the performance of real heat engines

This lecture we are going to cover:

- Practical Engine Cycles : Internal Combustion Engines
- Otto and Diesel Cycles : basic characteristics

Limitations of Practical Engines Over the Carnot Efficiency

What do we mean by a practical Engine ?

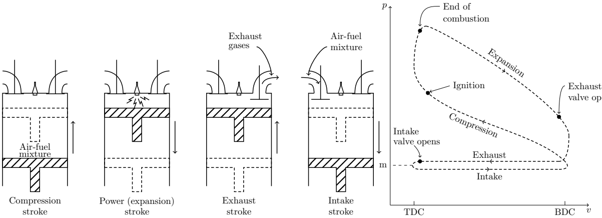
Other than the *isothermal* heat transfer process of the Carnot cycle in the *real* world..

- The hot reservoir temperature is limited by a number of factors : maximum combustion temperature possible, combustion temperature limitations due to combustion stability, emissions formation, mechanical failure of components at high temperatures
- The cold reservoir temperature limit is defined by ambient conditions
- Heat Transfer inefficiencies : heat not donated or lost at constant conditions usually
- Losses due to fluid mixing, turbulence (fluid viscous losses), mechanical losses.
- Mechanical losses (friction, crevice flows).

Given these losses, and the innate limitations of heat engine (Carnot) efficiency, practical internal combustion engines are actually very efficient.
Modern techniques have dramatically improved performance, fuel economy and emissions characteristics.
First we discuss petrol engines (Otto Cycle) and then the Diesel Cycle.

Otto Cycle / 4 stroke Spark Ignition Engines (1876)

The actual P-V path is quite complicated



- Compression stroke : from BDC to TDC (note clearance volume), both valves closed
- Power/expansion stroke : Ignition of compressed mixture (near constant volume process with pressure/temperature rise), expansion of combustion mixture
- Exhaust Stroke : Exhaust valve opens, hot gases pushed out of cylinder by stroke (> ~atm)
- Intake Stroke : exhaust valve closes, intake open, fresh cooler air charge sucked into cylinder (< ~atm), Injection of liquid fuel usually through intake port

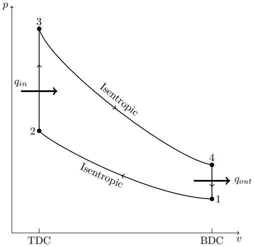
Exhaust/Intake strokes consume negligible power

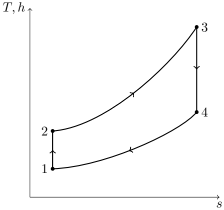
Otto Cycle Thermodynamics

Apply the *air standard* assumptions

It is a closed system so first law for any process is $q_{12} - w_{12} = e_{u,12}$ Heat transfer : $q_{in} = q_{23}$, $q_{out} = -q_{41}$

- Heat Addition: $q_{23} = C_v(T_3 - T_2)$
- Heat Rejection: $-q_{41} = C_v(T_4 - T_1)$
- $\eta_{th,Otto} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$
- Using adiabatic compression relations, eg, $pv^{\gamma} = C$
- $\frac{T_1}{T_2} = (\frac{v_2}{v_1})^{\gamma - 1}$, $\eta_{th,Otto} = 1 - \frac{1}{r^{\gamma - 1}}$
- Here r is the volume compression ratio $r = \frac{v_1}{v_2}$

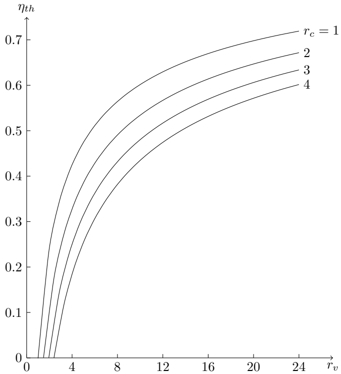




Otto Cycle Efficiency

Performance and Limits

- Ideal Otto efficiency a strong function of r for small r
- Real value will be less due to irreversibilities
- At large r the improvements are not so great
- Large r operation becomes difficult due to ‘knock’
- As the fuel mixture compresses it heats up and can auto-ignite (knocking), which impacts performance and causes engine damage.
- Lead used to be added to improve stability at larger r but phased out in the late 70’s due to toxic air pollution issues.
- Despite this modern autos, without anti-knock agents, outperform older leaded engine models significantly.
- Primarily due to advanced electronically controlled fuel injection systems.



Diesel Cycle/4 Stroke Compression Ignition engines (~1890)

Diesel (compression ignition) engines have the same basic processes as petrol (Otto) engines with a few key differences.

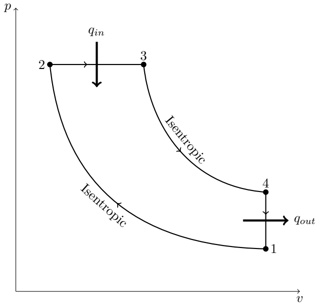
- Fuel is not injected during the intake stroke as in a petrol engine but injected near TDC.
- Heat addition is a constant pressure process
- Therefore *knock* is impossible during the compression stroke.
- Much larger compression ratios are possible, with improvements in thermal efficiency over petrol engines at lower ratios.
- There is no spark plug – the fuel auto-ignites when injected.
- Because the system is much less sensitive to early ignition, a wider and cheaper range of fuels can be used, from biodiesels, mineral oil to straight vegetable oil.

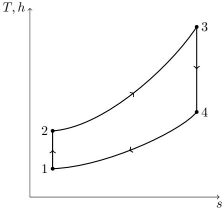
Diesel Cycle Thermodynamics

Apply the *air standard* assumptions

It is a closed system so first law for any process is $q_{12} - w_{12} = e_{u,12}$

- Heat Addition: $q_{23} = C_p(T_3 - T_2)$
- Heat Rejection: $-q_{41} = C_v(T_4 - T_1)$
- $\eta_{th,Diesel} = \frac{w_{net}}{Q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_4 - T_1}{\gamma(T_3 - T_2)}$
- Defining a“cutoff ratio”, $r_c = \frac{v_3}{v_2}$, the volume change during the combustion process,
- $\eta_{th,Diesel} = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{r_c^{\gamma} - 1}{\gamma(r_c - 1)}\right]$
- Where again r is the volume compression ratio $r = \frac{v_1}{v_2}$



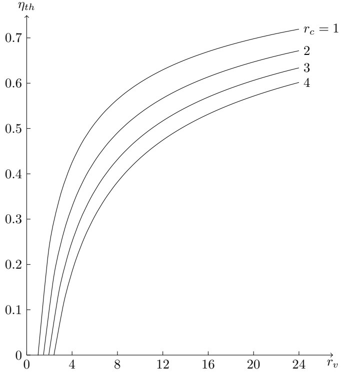


Otto and Diesel Cycle Comparison

Performance and Limitations

$\eta_{th,Diesel} = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{r_c^{\gamma}-1}{\gamma(r_c-1)}\right.$

- Diesel efficiency same as Otto efficiency with the exception of the [...] term
- ... >1 always, $\eta_{th,Otto} > \eta_{th,Diesel}$
- ... =1 for $r_c = 1$. Instantaneous combustion, like a spark, $\eta_{th,Otto} = \eta_{th,Diesel}$
- Although $\eta_{th,Otto} > \eta_{th,Diesel}$, Diesel engines run at much higher r, more efficient.
- Fuel is generally cheaper to produce and has a higher energy content per kg
- Powerplant of choice for large steady power output. Trucks, ships, back up power.
- Advanced engineering has enabled Diesel autos to approach petrol engine powerplant in terms of drivability.



Summary of this Lecture/Suggested Reading

Lecture Summary

- Practical Engine Cycles : Internal Combustion Engines
- Otto and Diesel Cycles : basic characteristics
- Key difference between Otto and Diesel cycles in the heat injection assumption.
- A Otto cycle is theoretically more efficient, the Diesel cycle gives better practical efficiency.

Suggested Further Reading

- Cengel and Boles,“Thermodynamics : An Engineering Approach”, Chapter 9
- Shrimpton section 3.9-3.12

Suggested Further Study

- Try chapter 7 questions, worked solution 7.9

Practical Engine Cycles : Gas Turbines

Last Lecture we covered:

- Practical Engine Cycles : Internal Combustion Engines
- Otto and Diesel Cycles : basic characteristics
- Key difference between Otto and Diesel cycles in the heat injection assumption.
- A Otto cycle is theoretically more efficient, the Diesel cycle gives better practical efficiency.

This lecture we are going to cover:

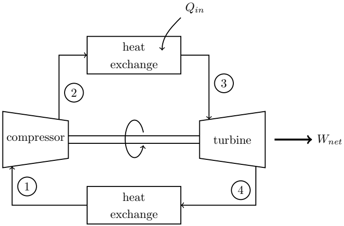
- The Brayton Cycle : Gas turbines (aircraft, stationary power)
- Approximation for a closed cycle system
- Thermal efficiency characteristics
- Discussion of inefficiencies
- Regeneration, Reheating, Intercooling

The Brayton Cycle/Gas turbines : Closed Cycle Approximation

Gas turbines usually operate on a open cycle where the turbine drives the compressor and provides the remainder of shaft work to the environment

Fresh air is drawn in, combustion fuel mixed, ignited and the exhaust gases discharged – in the case of a jet engine through a nozzle.

The turbine can be approximated as a closed cycle using air-standard assumptions



- the combustion process is replaced by a constant pressure heat addition
- the exhaust is replaced by a constant pressure heat rejection process.

The Brayton Cycle/Gas turbines : Processes

Two constant Pressure heat processes

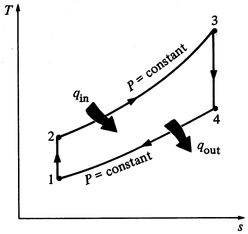
- Process 12 : Isentropic compression (compressor)
- Process 23 : Constant pressure heat addition (combustor)
- Process 34 : Isentropic expansion (turbine)
- Process 41 : Constant pressure heat rejection (exhaust)

- Heat Addition: $q_{23} = C_p(T_3 - T_2)$
- Heat Rejection: $-q_{41} = C_p(T_4 - T_1)$

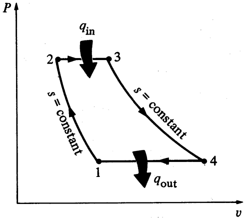
$$\eta_{th,Brayton} = \frac{w_{net}}{Q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_4 - T_1}{(T_3 - T_2)}$$

Use isentropic relations to convert to a pressure ratio, $\frac{T_1}{T_2} = (\frac{P_2}{P_1})^{\frac{1-\gamma}{\gamma}}$

$\eta_{th,Brayton} = 1 - \frac{1}{r_p^{(\gamma-1)/\gamma}}$ where $r_p = \frac{P_2}{P_1}$



(a) T-s diagram

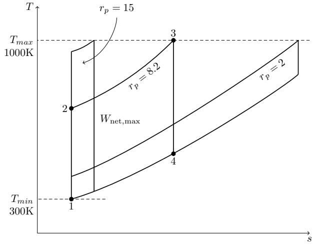


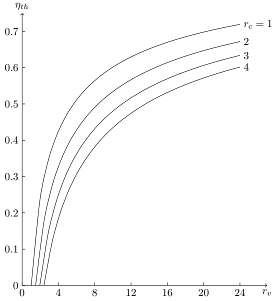
(b) P-v diagram

Brayton Cycle Performance and Limitations

Same as the Otto Cycle

- Efficiency increases with pressure ratio.
- The limiting cycle is T_3, the hot combustion gases going into the turbine.
- However for fixed turbine inlet temperature (T_3) the work obtained from the turbine increases at small r_p, to an optimum, before finally decreasing again at high r_p.
- Air supplies oxygen for combustion but also provides cooling to combustion chamber surfaces.
- The amount of energy the compressor takes from the turbine (back-work) is usually high, and often steam (Rankine - not covered here) cycles are used which have much smaller back work amount.

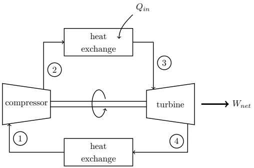


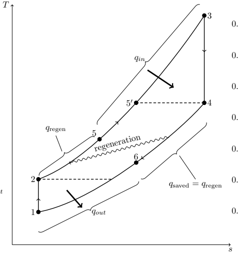


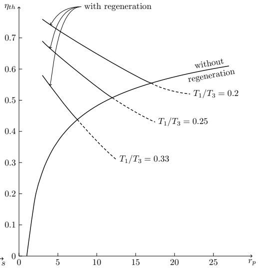
Brayton Cycle with Regeneration

Increasing efficiency at low pressure ratio

- Temperature leaving the turbine, T_4, is pure wasted heat - use preheat the inlet feed (T_5).
- If the regenerator was 100% efficient, $T_5 = T_4$, however a more economical value is ~0.7.
- With this value, as noted in the T-S diagram, q_{out} is reduced (by q_{saved}) and q_{in} is reduced by q_{regen}.
- Regeneration works best when the pressure ratio is low. Note the dotted lines on the isotherms of the efficiency curve is where compressor stream is heating the turbine exit!





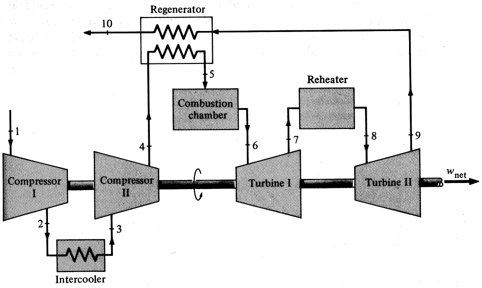


Brayton Cycle with Intercooling, reheating and regeneration.

Multiple small stages better than one big one

The net work of gas-turbine cycle can be improved by reducing the back-work done by the compressor or increasing the output of the turbine. This is achieved by :

- Compressor : Using Intercooling - constant pressure heat removal. This increases the gas density and the compression efficiency.
- Turbine : Using Reheating - constant pressure heat addition. This increases the temperature entering the turbine, and increasing the exit temperature. (This also benefits the regenerator!).



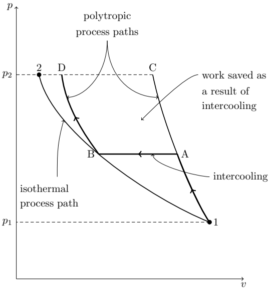
Intercooling operation and T-S Diagram

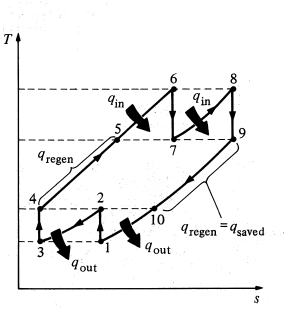
Easier to compress a cool gas..

The Intercooling staged versus un-staged compression, where the objective is to get the pressure from P_1 to P_2.

Staged Brayton Cycle

The T-S diagram shows the complete cycle for the system shown on the previous slide. We can see the intercooling and the reheating enable significantly more area to be enclosed.
Also notice the reduction in energy taken from the hot reservoir (q_{in}) and donated to the cold (q_{out}) .





Summary of this Lecture/Suggested Reading

Lecture Summary

- Brayton Cycle Introduction
- Two constant pressure heat addition/rejection processes
- Practical methods to improve efficiency - regeneration intercooling/reheating

Suggested Further Reading

- Cengel and Boles,“Thermodynamics : An Engineering Approach”, Chapter 9
- Shrimpton section 3.13

Suggested Further Study

- Try chapter 7 questions.

Lecture 12: Conservation Laws

Last Lecture we covered:

- Brayton Cycle Introduction
- Two constant pressure heat addition/rejection processes
- Practical methods to improve efficiency - regeneration intercooling/reheating

This lecture we are going to cover:

- Thermofluid variables - forms of energy, Joule’s Experiment.
- Basic Conservation Laws : Mass, Momentum and Energy.
- Lagrangian and Eulerian Viewpoints
- Systems and Control Volumes
- The material derivative
- The Reynolds Transport Theorem.

Conservation Principles : Extensive and Intensive quantities.

Extensive Properties [A]

An extensive quantity is a quantity that is a function of how much if it there is.
Mass, Momentum and Energy are thus extensive quantities.

Intensive Properties [a]

When writing conservation laws for these properties, we would prefer to write general equations, that are *specific*. These are intensive properties

Mass ($A = m$)	Density ($a = l$)
Momentum ($A = mU$)	Velocity ($a = U$)
Internal Energy ($A = E_u$)	Specific Internal Energy ($a = e_u$)

For any *extensive* property A, its corresponding *intensive* property a may be defined,
$A = \int \rho a \partial V \approx \rho a \Delta V$.
In this module, we use UPPER case for *extensive*, and lower case for *intensive*.

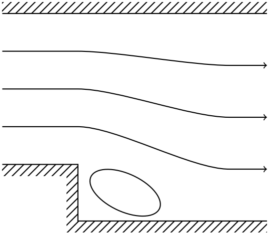
Example

- V and v (also 1/ ρ!), S and s etc.

Fundamental Conservation Laws.

- Conservation of Mass ...
- Conservation of Momentum: Newton's 2^{nd} Law: $F = ma$.
- Conservation of Energy: First Law of thermodynamics.

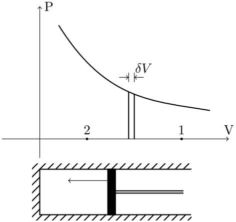
- Our fundamental conservation laws are based around a fixed mass.
- That mass can be moving/changing shape (apples and balloons..)



Usually we want to work out how much fluid moves through a device (a jet engine say) and so we need conservation laws based around a fixed volume through which a mass can move.
We therefore need to devise a 100% way to convert conservation laws to our fixed mass framework to a fixed volume framework

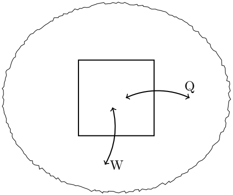
Fixed Mass Conservation : Systems and Environments.

Fixed Mass Features



- A fixed mass is a continuous mass of fluid that always contains the same mass (i.e. the same molecules). The shape/volume can change.
- The information associated with the fixed mass moves with it, e.g U(t) of an apple. This is a Lagrangian description) of a fixed mass.
- The ’fixed mass’ is known as a system. The continuum it sits in is known as the environment
- *Only* Energy crosses the system boundary.

Example



- Heating a gas in a box is a system process.
- Heating a gas in a flexible box so it expands is a system process.
- The gas in a flexible box can do some work on the environment.

Forms of Energy, and Energy Transfer

Thermofluid operations that occur in systems are called closed processes and these are considered in this term of this course.
Because engines work on cycles, they can be approximated by a system analysis (the intake/exhaust processes can be approximated as heat transfers) - they are also dealt with with this method.

Forms of Energy

Potential Energy : $E_g = mgx$
Kinetic Energy : $E_k = \frac{1}{2}mU^2$
Displacement Work : Fx
Thermal (Internal) Energy : $E_u = mC_V T$

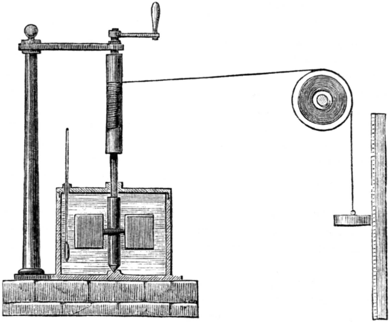
Example

... of Energy Transfers

- Water Flowing over a waterfall: Potential → Kinetic Energy.
- Heating a gas in a rigid insulated container. Heat transfer → Internal energy.
- Heating a gas in a cylinder-piston assembly. Heat Transfer → Internal Energy + Displacement Work.

Joule's Work → Heat Experiment.

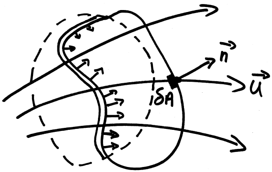
- Joule developed an experiment to show work can be converted to heat.
- He put a paddle wheel into an insulated container and powered it by a weight descending on a pulley.
- He showed that the temperature of the fluid rose when work was exerted on it.
- More detail here:
- `http://en.wikipedia.org/wiki/Jame`

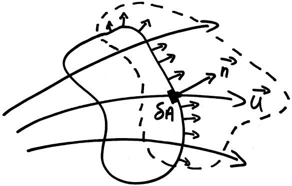


Lagrangian and Eulerian volumes in a flow.

Relating a system to a control volume : Reminders

- Our fundamental conservation laws use a fixed mass basis. The information tracks the mass.
- This is useful for Apples and Engines, but not generally.
- Non-fundamental conservations (where we don't track the same molecules) much more useful for us.
- We need a way to translate the this information without any error.





The plan is...

- Relate the Eulerian frame to the Lagrangian frame for a point quantity.
- Do the same for a fixed mass to a fixed volume.

Eulerian and Lagrangian viewpoints

Imagine you are sitting on a river bank, recording the temperature of the water two different ways, and at some point in time they measure the same T.

Eulerian and Lagrangian methods

- Method 1 Imagine you have a grid of sensors in the river, choose your sensor there and measure $T(\overrightarrow{x}, t)$.
- Method 2 You place your sensor on a float, moving with the velocity of the river, and measure $T(\overrightarrow{x}(t), t)$

(In the Lagrangian sense), we would like to track how the floats temperature changes with time

- We assume an *infinitesimal* time interval, δt.
- In that time $T(\overrightarrow{x}, t)$ has changed to $T(\overrightarrow{x} + \overrightarrow{u}\delta t, t + \delta t)$.
- We define how the temperature changes on the float.

$$\frac{D}{Dt}T(\overrightarrow{x}, t) = Lim_{\delta t \to 0} \frac{T(\overrightarrow{x} + \overrightarrow{u}\delta t, t+\delta t) - T(\overrightarrow{x}, t)}{\delta t}$$

The key point to remember here is that the velocity of the float $\overrightarrow{u} = \frac{\partial \overrightarrow{x}}{\partial t}$.
Note that this is the Eulerian partial derivative, $\overrightarrow{u}(\overrightarrow{x}, t)$

The Material Derivative

Recall our float temperature change is defined :
$\frac{D}{Dt}T(\overrightarrow{x},t) = Lim_{\delta t \to 0}\frac{T(\overrightarrow{x}+\overrightarrow{u}\delta t, t+\delta t)-T(\overrightarrow{x},t)}{\delta t}$

Material Derivative in 1D - Taylor series

- Expanding $T(x,t)$ in time requires us to take account of the position change :
- $T(x(t)+u\delta t, t+\delta t) = T(x(t),t) + \delta t\frac{\partial T(x,t)}{\partial t} + \delta x\frac{\partial T(x,t)}{\partial x}$....or.....
- $\frac{T(x(t)+u\delta t)-T(x(t),t)}{\delta t} = \frac{\partial T(x,t)}{\partial t} + u\frac{\partial T(x,t)}{\partial x}$
- Notice that the RHS is all Eulerian quantities. So in 1-D : $\frac{DT}{Dt} = \frac{\partial T}{\partial t} + U\frac{\partial T}{\partial x}$

Material Derivative in 3D - Taylor series again

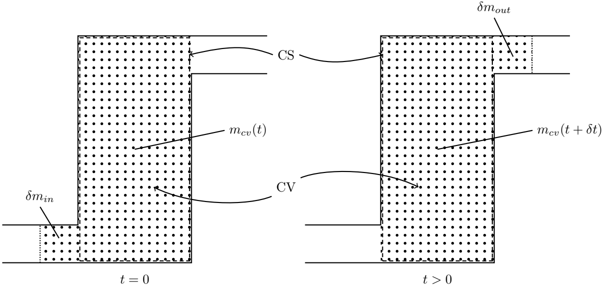
- $T(y(t),t) + \delta t\frac{\partial T(y,t)}{\partial t} + \delta y\frac{\partial T(y,t)}{\partial y}$
- $T(z(t),t) + \delta t\frac{\partial T(z,t)}{\partial t} + \delta z\frac{\partial T(z,t)}{\partial z}$
- $T(\overrightarrow{x}(t),t) + \delta t\frac{\partial T(\overrightarrow{x},t)}{\partial t} + \delta\overrightarrow{x}\frac{\partial T(\overrightarrow{x},t)}{\partial\overrightarrow{x}}$
- so the Material Derivative is : $\frac{DT}{Dt} = \frac{\partial T}{\partial t} + U\frac{\partial T}{\partial x} + V\frac{\partial T}{\partial y} + W\frac{\partial T}{\partial z}$

The LHS is how T changes in the Lagrangian sense (on the float) and the RHS is how ϕ changes in the Eulerian sense (from the sensor position).

Lagrangian to Eulerian approaches for a finite mass/volume.

The Reynolds Transport Theorem

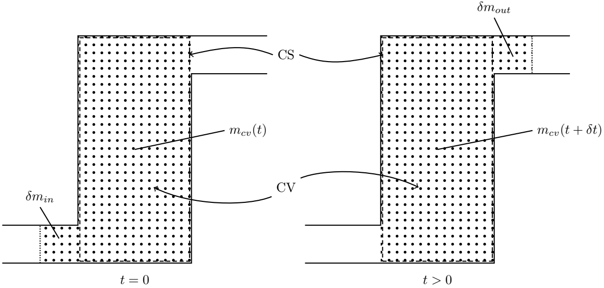
- The Reynolds Transport Theorem is derived by considering the rate of change of any extensive variable of a system as it passes through a fixed control volume.
- We will use mass as our *extensive* variable (density is the *intensive* equivalent).
- Notice also in the example that the volume of the system is slightly larger than the volume of the control volume - at time 0 a little bit has not yet flowed in.



In words our problem is :

"rate of change of A in system" = "rate of change of a in CV" + "net outflow of a through CV surface"

Reynolds Transport Theorem (1/2)



System Mass Balance

The mass of the system at time 0 is: $m_{sys}(0) = m_{CV}(0) + \delta m_{in}$
And at time δt: $m_{sys}(\delta t) = m_{CV}(\delta t) + \delta m_{out}$

Mass change of the system from the material derivative: $\frac{Dm_{sys}}{Dt} = \lim_{\delta t \to 0} \frac{m_{sys}(\delta t) - m_{sys}(0)}{\delta t}$

Control volume mass balance

Substituting in the above definitions: $\frac{Dm_{sys}}{Dt} = \lim_{\delta t \to 0} \frac{m_{CV}(\delta t) + \delta m_{out} - m_{CV}(0) - \delta m_{in}}{\delta t}$

Re-arranged: $\frac{Dm_{sys}}{Dt} = \lim_{\delta t \to 0} \frac{m_{CV}(\delta t) - m_{CV}(0)}{\delta t} + \lim_{\delta t \to 0} \frac{\delta m_{out}}{\delta t} - \lim_{\delta t \to 0} \frac{\delta m_{in}}{\delta t}$

Reynolds Transport Theorem (2/2)

From the previous slide...

$$\frac{Dm_{sys}}{Dt} = \lim_{\delta t\to 0} \frac{m_{CV}(\delta t) - m_{CV}(0)}{\delta t} + \lim_{\delta t\to 0} \frac{\delta m_{out}}{\delta t} - \lim_{\delta t\to 0} \frac{-\delta m_{in}}{\delta t}$$

- 1st term RHS is the rate of change of mass inside the CV, and is thus an Eulerian quantity : $\frac{\partial m}{\partial t}$.
- Converting this to an intensive quantity: $\frac{\partial m}{\partial t} = \frac{\partial}{\partial t} \int \rho \partial V$.
- 2^{nd} and 3^{rd} terms are the net rate of mass flow rate into the CV.
- If the flow was uniform, then the mass flow rate out of the CV would be $\rho U_n A$, where the U_n is the speed of the flow normal to the CV face.
- Generalising, the net mass flow into the CV, using Gauss's Law: $\int_A \rho(\overrightarrow{U} \bullet \overrightarrow{n})\partial A$.

Which gives us the final form of the Reynolds Transport theorem:

- $\frac{Dm_{sys}}{Dt} = \frac{\partial}{\partial t} \int \rho \partial V + \int \rho(\overrightarrow{U} \bullet \overrightarrow{n})\partial A$.
- On the RHS, our fixed mass, on the LHS our fixed volume

In words

"rate of change of A in system" = "rate of change of a in CV" + "net outflow of a through CV surface"

Cool Videos..

Complex, subtle lecture - text books are not much help. Try these...

- https://www.youtube.com/watch?v=zUaD-GMARrA <= Start here !
- https://www.youtube.com/watch?v=mdN8OOkx2ko <= then this!
- https://www.youtube.com/watch?v=BLVY69SYVBQ <= or this
- https://www.youtube.com/watch?v=G1kILyV0GDM <= Finally this !

Summary and Suggested Reading

Lecture Summary

- Thermofluid variables - forms of energy, Joule's Experiment.
- Basic Conservation Laws : Mass, Momentum and Energy.
- Lagrangian and Eulerian Viewpoints
- Systems and Control Volumes
- The material derivative
- The Reynolds Transport Theorem.

Suggested Further Reading

- Crowe et al, Chapter 5.
- Cengel and Boles, Section 1.3,5.2, Section 5.1.
- Shrimpton, Section 2.2-2.3

Suggested Further Study

- Workbook Chapter 2 can now be completed
- worked solution 3.17

Lecture 13: Mass Conservation.

Last Lecture we covered:

- Thermofluid variables - forms of energy, Joule's Experiment.
- Basic Conservation Laws : Mass, Momentum and Energy.
- Lagrangian and Eulerian Viewpoints
- Systems and Control Volumes
- The material derivative
- The Reynolds Transport Theorem.

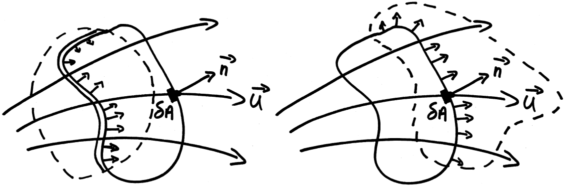
This lecture we are going to cover:

- Application of the Reynolds Transport theorem.
- Mass Flow across a surface.
- Mass Conservation.
- A few examples.

As an Engineer, we need an Eulerian Conservation of Mass

We already have a Lagrangian Version..

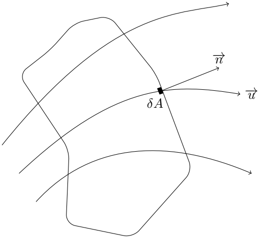
- $m_{sys} = 0$ or $Dm_{sys}/Dt = 0$
- We can *automatically* get an Eulerian Version using the Reynolds Transport Theorem.
- $\frac{Dm_{sys}}{Dt} = \frac{\partial}{\partial t}\int \rho dV + \int \rho(\overrightarrow{U} \bullet \overrightarrow{n})dA.$
- On the RHS, our fixed mass, on the LHS our fixed volume. So..
- $\frac{\partial}{\partial t}\int \rho dV + \int \rho(\overrightarrow{U} \bullet \overrightarrow{n})dA = 0.$



Reynolds Transport Theorem is extremely powerful because

- We can use any variable
- ....but it does not really help us *understand*

Direct Eulerian Mass Conservation (1/2)



Imagine we have some arbitrary steady 2D flowfield, defined by a set of streamlines.

Now suppose we draw an imaginary area somewhere in this flowfield.

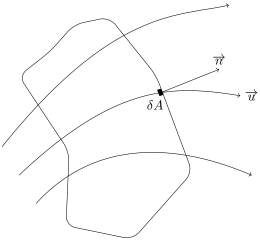
Now suppose we want to ensure mass is conserved within this area (volume in 3D)

We do this by defining the mass flow through a tiny elemental area δA.

At this tiny elemental area δA , the velocity vector, $\vec{U}$ passes through it at a different angle to the unit normal of the area, $\vec{n}$, a vector of unit magnitude.

Conceptually, the mass flow through the area element is due only to the normal component of the velocity.

Direct Eulerian Mass Conservation (2/2)



What we want to do is find out how much of $\overrightarrow{U}$ is going in the $\overrightarrow{n}$ direction.

From simple trigonometry the normal component is $|\overrightarrow{U}|cos\theta$.

General dot product definition:

$\overrightarrow{a} \bullet \overrightarrow{b} = a_1b_1 + a_2b_2 + a_3b_3$ = $|\overrightarrow{a}||\overrightarrow{b}|cos\theta$.

In this case: $\overrightarrow{U} \bullet \overrightarrow{n} = |\overrightarrow{U}|cos\theta$.

So the mass flow through the small element is $\delta\dot{m} = \rho(\overrightarrow{U} \bullet \overrightarrow{n})dA$.

Sum all elements, to give a mass conservation over the entire surface (volume). This will be positive if there is a net loss of mass in the enclosed volume:
$\int \delta\dot{m} = \int \rho(\overrightarrow{U} \bullet \overrightarrow{n})dA$.

For steady flows the net mass flow must be zero, i.e.: $\int \rho(\overrightarrow{U} \bullet \overrightarrow{n})dA = 0$.

Comparing Our Approaches

RTT versus Eulerian

- Eulerian Approach : $\int \rho(\overrightarrow{U} \bullet \overrightarrow{n})dA = 0$
- RTT Approach : $\frac{\partial}{\partial t} \int \rho dV + \int \rho(\overrightarrow{U} \bullet \overrightarrow{n})dA = 0$

Viscous Fluids

We usually have to apply the full form, because of the varying velocity in the boundary layer.

Inviscid Fluids

For steady uniform flow, expressing normal velocities directly:
$\sum_{inlets} \rho U_n A = \sum_{outlets} \rho U_n A$.
For incompressible fluids, the volume conservation equation:
$\sum_{inlets} U_n A = \sum_{outlets} U_n A$.

Summary and Suggested Reading

Lecture Summary

- Defined an Eulerian mass conservation equation.
- Defined some simple forms for an inviscid fluid.
- Covered some examples.

Suggested Further Reading

- Crowe et al. 'Engineering Fluid Mechanics', 9^{th} ed. SI version, Chapter 5.
- Shrimpton, Section 2.2.9-2.2.20, 2.3, 4.5-4.6

Suggested Further Study

- Workbook Chapter 9 mass conservation parts (do a few..)
- solutions 9.13, 9.16

Lecture 14: (Intro to) Momentum Conservation.

Last Lecture we covered:

- Defined an Eulerian mass conservation equation.
- Defined some simple forms for an inviscid fluid.
- Covered some examples.

This lecture we are going to cover:

- Newton's Second Law, Definition of Momentum, relation to acceleration and force.
- Use the Reynolds Transport Theorem.
- Consider a way to visualise this.
- Inviscid forms.
- Examples.

Mass, Velocity, Momentum, Force and Acceleration.

Scalar and Vector Quantities

- Mass (extensive) and its intensive quantity (density - mass per unit volume) are scalars.
- Momentum (extensive) and its intensive quantity (velocity - momentum per unit mass) are vectors.
- Force (extensive) and its intensive quantity (acceleration - force per unit mass) are all vectors.

So when we write a conservation equation for mass (or density) we write one equation. When we write a conservation equation for momentum (or velocity) we write three equations, one for each direction.

- Momentum in the x-direction is $mU_x = M_x$, the momentum vector is $m\overrightarrow{U} = \overrightarrow{M}$.
- Rate of change of momentum is $\frac{d}{dt}(mU_x) = F_x = ma_x$.
- So Newton's Second Law is a rate of momentum change equation.

Before we get onto deriving a momentum conservation equation - remind ourselves of mass conservation.

To find the mass flow through a surface element.

What we want to do is find out how much of $\overrightarrow{U}$ is going in the $\overrightarrow{n}$ direction.

From simple trigonometry normal component $|\overrightarrow{U}|cos\theta$

General dot product definition:

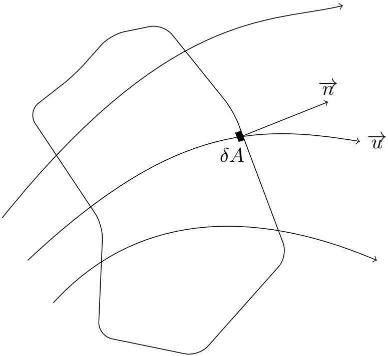
$$\overrightarrow{a} \bullet \overrightarrow{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\overrightarrow{a}||\overrightarrow{b}|cos\theta$$

In this case: $\overrightarrow{U} \bullet \overrightarrow{n} = |\overrightarrow{U}|cos\theta$

So the mass flow through the small element is $\delta\dot{m} = \rho(\overrightarrow{U} \bullet \overrightarrow{n})dA$

All elements, and mass conservation over the entire surface (volume) $\int \delta\dot{m} = \int \rho(\overrightarrow{U} \bullet \overrightarrow{n})dA$.

For incompressible fluids the net mass flow must be zero, i.e. $\int \rho(\overrightarrow{U} \bullet \overrightarrow{n})dA = 0$.



Eulerian Conservation of Momentum

We already have a Lagrangian Version..

- $\overrightarrow{F} = m\overrightarrow{a}$
- We can *automatically* get an Eulerian Version using the Reynolds Transport Theorem
- $\overrightarrow{F} = \frac{\partial}{dt}\int \rho \overrightarrow{U} dV + \int \rho \overrightarrow{U}(\overrightarrow{U} \bullet \overrightarrow{n})dA.$

Reynolds Transport Theorem is extremely powerful because

- We can use any variable
- ....but it does not really help us *understand*

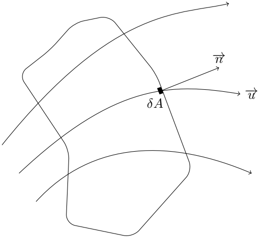
The velocity field 'carries' mass/mom information....

Flow field 'carries' density

Density = mass per unit volume = specific mass The mass flow through the small element is:

$$\delta \dot{m} = \rho(\overrightarrow{U} \bullet \overrightarrow{n})\delta A$$

Velocity field *transporting* density (ρ) information in/out of each surface element of the entire volume.



Flow field 'carries' velocity

Momentum flow in the x-direction through the element would be:

$$\delta \dot{M}_x = \rho U_x(\overrightarrow{U} \bullet \overrightarrow{n})\delta A$$

$$[kgm^{-3}.m^2 s^{-2}.m^2 = kg.ms^{-2} = N]$$

Here, imagine the velocity field is carrying a small packet of x-momentum in/out of the volume.

Conservation of Momentum Equation.

Conservation of momentum is...

- 'Rate of accumulation of momentum in the volume + net rate of momentum leaving the volume = force applied to the surface of the volume'
- $\frac{\delta}{\delta t}(\int \rho U_x \delta V) + \int \rho U_x (\overrightarrow{U} \bullet \overrightarrow{n}) \delta A = \sum F_x$

Forces

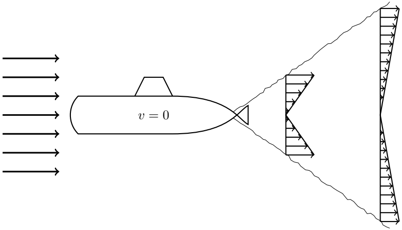
Pressure qradient, gravity (acceleration), viscous forces, exotics.

Simplifications

- We will only consider steady flows in this module : $\frac{\partial}{\partial t}(\int \rho U_x dV) = 0$
- For inviscid flows with constant flow over inlet/outlet $\dot{M}_x = \dot{m} U_x$ and so,
- $\sum_{out} \dot{M}_x - \sum_{in} \dot{M}_x = \sum F_x$
- one inlet+outlet $\dot{m}(U_{out,x} - U_{in,x}) = \sum F_x$
- no flow (statics !) $\sum F_x = 0$

Moving Control volumes.

- The vast majority of the time our control volume is fixed in space, but there are occasions when we want to define a moving control volume .
- Remember to convert the answer back into the correct frame of reference when you have done the question. !



Example

- Let us say we wish to define the wake behind a submerged submarine moving through the ocean.
- If instead we change the frame of reference, i.e. $V = -U$, then our submarine is stationary and the ocean flows past it.
- The wake is now stationary.

Summary and Suggested Reading

Lecture Summary

- Reminded ourselves that mass and density are scalar extensive and intensive quantities.
- Invoked Newton’s 2nd Law and unsteady conditions to define a general momentum conservation law.
- Simple Forms of the Momentum Equation

Suggested Further Reading

- Crowe et al. ’Engineering Fluid Mechanics’, 9^{th} ed. SI version, Section 6.1-6.4, 6.6.
- Shrimpton, Section 2.2.9-2.2.20, 2.3, 4.8

Suggested Further Study

- Try chapter 10 questions (a few), solutions 10.3 (and with a turn)

Lecture 15: Fluid Statics : The hydrostatic equation

Last Lecture we covered:

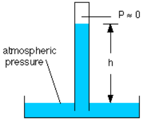
- Momentum Conservation
- Simple forms of the equation
- Some examples

This lecture we are going to cover:

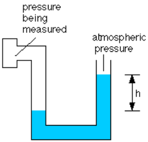
- The hydrostatic equation – how static pressure varies in a column of a fluid parallel to the direction of gravity.
- Simplifications of this if the fluid is incompressible.

Problem Outline

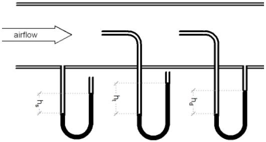
Examples



closed-tube manometer
(a simple barometer)



open-tube manometer



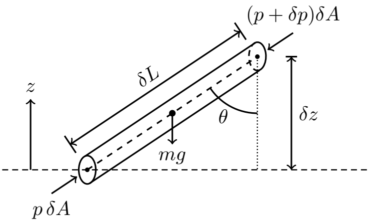
The plan is..

- We are going to relate the pressure change in height to the weight of a stationary liquid in a small cylinder of fluid.
- This is an application of the momentum equation, where $\sum F_z = 0$
- At some z we know the pressure p, what is the pressure at some height Δz, along some length ΔL, at some angle theta, in a fluid of some density...
- We want to find Δp in terms of Δz

Assumptions and Equation Derivation

We choose a small element of fluid

The weight acts in the z direction, the pressure force acts in the L direction.
Need to resolve the pressure force in the same direction.

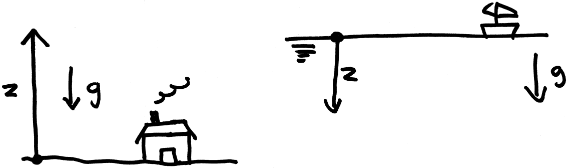


- The fluid element is stationary, therefore the forces must balance $\sum F_L = 0$.
- The weight acting in the z direction is $-mg = -\rho V g = -\rho \delta L \delta A g$.
- In the L direction the weight is $mgcos\theta = \rho \delta L \delta A g cos\theta$
- The pressure force acting in the L direction is $(p+\delta p)\delta A - p\delta A = \delta p \delta A$
- Equating forces: $-\rho \delta L \delta A g cos\theta = \delta p \delta A$

Since $\delta z = \delta L cos\theta$ and in the limit of δz , $\delta A \to 0$, $\frac{dp}{dz} = -\rho g$

A word about sign convention...

It pays to think about the sign convention : Does pressure increase with height/depth ???



- $\frac{dp}{dz} = -\rho g$ or $\frac{dp}{dz} = \rho g$

Notes and Integral Forms

Our *master* equation is..

$\frac{dp}{dz} = -\rho g$

- “the rate of pressure change with direction is negative”
- For dz=0, dp=0, as long as density is constant : same pressure at a constant z plane

Integration yields....

- $p = -\int \rho g dz$
- Finding how p varies with height requires knowing how ρ varies with height

For incompressible fluids..

- $p = -\rho g z + const$
- constant depends on the pressure at z=0. If g and the z-axis are pointing in the same direction then..
- $p = p_{atm} - \rho g z$

Notes.

- Here $\rho g z$ is the gauge pressure, linear increase in pressure with depth
- Dividing by ρg gives an equation with dimensions of $[L] - eg$ pressure in terms of mmHg

Examples : Density variations in the hydrostatic equation

Integration of $p = -\int \rho g dz$ requires knowledge of how density varies with z.

Two relevant examples

Density is a function of temperature, and temperature is a function of height.

This might happen in a liquid...

- $\rho(T) = \rho_0 - \alpha T$ and $T = T_0 + \beta z$
- $p = -g \int \rho_0 - \alpha(T_0 + \beta z)dz = -g((\rho_0 - \alpha T_0)z - \alpha\beta z^2/2) + c$

Density is a function of pressure but not temperature

this is a typical for the atmosphere.....

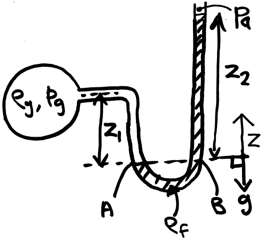
- $p = \rho RT$
- $\frac{dp}{dz} = -\rho g = -\frac{pg}{RT}$, so $\frac{dp}{p} = -\frac{g}{R}\frac{dz}{T}$, and remember T is constant.
- $\int \frac{dp}{p} = -\int \frac{g}{RT} dz$. Integrate this and you get.. $\ln(\frac{p}{p_0}) = -\frac{gz}{RT}$ where p_0 is p at z=0.

Pressure measurement in a U-tube manometer to measure p_g

Example

Level A-B must be at the same pressure (constant z)

- $(1) : p_A = p_g + \rho_g g z_1$
- $(2) : p_B = p_A + \rho_f g z_2$
- $p_A = p_B$
- $Eqn(1) - Eqn(2)$
- $p_{gauge} = p_g - p_a = (\rho_f z_2 - \rho_g z_1) g$
- If $\rho_g \ll \rho_f$, then $p_{gauge} = \rho_f g z_2$



Summary of this Lecture/Suggested Reading

Lecture Summary

- The use of a column (or head) of fluid to represent a pressure using a manometer
- Defined the hydrostatic equation in differential and integral form
- Highlighted the sign convention, how you have to think about how does pressure change with height/depth.
- Discussed a simplification if the fluid may be considered incompressible, and methods to describe the pressure change if it is not

Suggested Further Reading

- Crowe et al. ‘Engineering Fluid Mechanics’, 9_{th} ed. SI version. Section 3.1-3.2
- Shrimpton section 4.9.1.

Suggested Further Study

- Try Chapter 8 questions, worked solution 8.13

Lecture 16: Hydrostatic Forces

Last Lecture we covered:

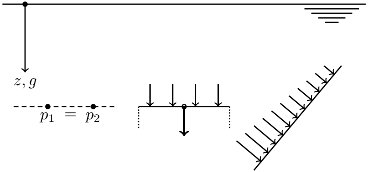
- The use of a column (or head) of fluid to represent a pressure using a manometer
- Defined the hydrostatic equation in differential and integral form
- Highlighted the sign convention, how you have to think about how does pressure change with height/depth.
- Discussed a simplification if the fluid may be considered incompressible, and methods to describe the pressure change if it is not

This lecture we are going to cover:

- Hydrostatic force distributions on submerged surfaces
- Integral measures of these force distributions acting through a single point on this submerged surface, the location of this integral force.
- Definitions of *centre of area* and *centre of pressure*
- Balancing this with a moment from a hinge on a gate (for instance).
- Applying this to curved surfaces

Pressure and force *direction*

Pressure is a scalar, the area is the vector defining the force direction



- Pressure is force per unit area, $\delta F = p\delta A$
- Pressure acts equally in all directions.
- It is constant at a given z where the z axis is parallel to the direction of the gravity.
- Pressure is a function of z (for constant density fluids a linear function)
- The direction of the surface (the normal to that surface) defines the direction that the force acts.

The horizontal case is easy

$\delta F = p\delta A$, so $\delta F = (\rho g z)\delta A$ so $F = (\rho g z)A$

Force and centre of area on an arbitrary inclined, arbitrary shaped, planar surface.

Finding the force

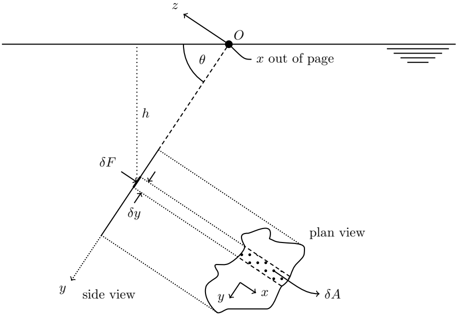
- $\delta F = p\delta A = (\rho g h)\delta A = (\rho g y sin\theta)\delta A$
- $F = \int \rho g y sin\theta \delta A = \rho g sin\theta \int y \delta A$

Definition of the 1_{st} moment of area:

- $\int y dA = \bar{y}A$
- $F = \rho g A \bar{y} sin\theta = \rho g A \bar{h}$

Note:

- F is the force on the surface
- $\bar{h}$ is the depth of the area centroid below the surface
- $P = F/A$ is the pressure at $\bar{h}$, divided by the area.

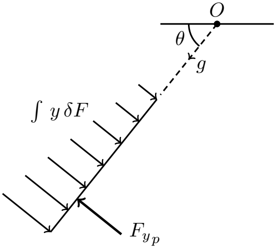


But where does this force act? *at the centre of pressure*

Centre of Pressure : The point where all the force can be said to act

Take moments about point O

- $\delta M_{aclock} = y\delta F = yp\delta A = y(\rho g h)\delta A$
- = $\delta M_{aclock} = \rho g y^2 sin\theta \delta A$
- $M_{aclock} = \int \delta M_{aclock} = \rho g sin\theta \int y^2 \delta A$
- $M_{clock} = F y_p$
- From previous slide,
  $F = \rho g A \bar{y} sin\theta = \rho g A \bar{h}$



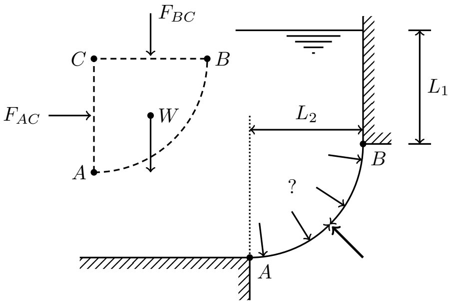
Therefore..

- $F y_p = \rho g sin\theta \int y^2 \delta A$
- A single location where all the force, distributed over the plate can be said to act.

Forces on curved gates : Force components

It is easier to sum forces on a free body diagram.

- In effect, we are doing our previous analysis twice, one for each coordinate direction.



Here, assume a circular gate, unit width. Summing forces in the horizontal (x) and vertical directions (y) gives

$$F_x = F_{AC} = \int pdA = \rho g \int ydy = \rho g [\frac{y^2}{2}] = \rho g \frac{L_2(2L_1+L_2)}{2}$$

$$F_y = W + F_{CB} = \rho g \frac{\pi L_2^2}{4} + \rho g L_1 L_2$$

The magnitude of the force on the gate is $F = (F_x^2 + F_y^2)^{1/2}$

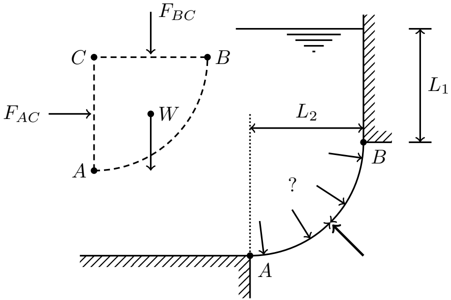
Forces on curved gates : Centres of pressure - horizontal force

Centre of pressure for the vertical direction is

$F_x y_p = \int p y dA = \rho g \int y^2 dy = \rho g[\frac{y^3}{3}] = \rho g[\frac{(L_1+L_2)^3 - L_1^3}{3}]$

Since $F_x = \rho g \frac{L_2(2L_1+L_2)}{2}$

So, $y_p = \frac{2}{3}[\frac{3L_1^2+3L_1L_2+L_2^2}{2L_1+L_2}]$



Forces on curved gates : Centres of pressure - vertical force

The centre of pressure for the horizontal direction is found by taking moments about C

$F_y x_p = F_{CB}\frac{L_2}{2} + W\bar{x}$

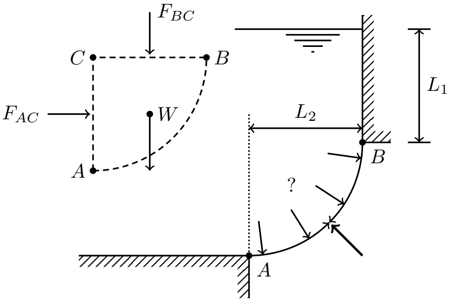
$\bar{x}$ is the distance from C to the centroid of area of a hemisphere.

In general this is $\bar{x} = \frac{\int x dA}{A}$.

It is easier to do in cylindrical cords.

The answer is $\bar{x} = \frac{4L_2}{3\pi}$

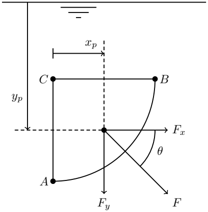
Therefore $x_p = \frac{1}{F_y}(\rho g \frac{\pi L_2^3}{8} + \frac{4\rho g L_1 L_2^2}{3\pi})$



Forces on curved gates : Magnitude and Direction of Resultant force.

We now have all the information we need :

- Magnitude and depth at which horizontal force acts F_x, y_p
- Magnitude and displacement at which vertical force acts F_y, x_p
- Resultant force Magnitude: $F = (F_x^2 + F_y^2)^{1/2}$
- Resultant direction: $tan\theta = F_x/F_y$



Key point about this analysis is that you do not find the curved surface force, you find the resultant force arising from forces acting on a body of fluid next to it. This means this analysis applies to any shape.

Summary of this Lecture/Suggested Reading

Lecture Summary

- Hydrostatic force distributions on submerged surfaces.
- Integral measures of these force distributions acting through a single point, the direction of this integral force.
- Balancing this with a moment from a hinge on a gate.
- Using free body diagrams to work out the forces on curved gates.

Suggested Further Reading

- Crowe et al. ‘Engineering Fluid Mechanics’, 9th ed. SI version,Section 3.4
- Shrimpton, section 4.9.2-4.9.3

Suggested Further Study

- Try chapter 8 questions.

Lecture 17: Buoyancy Forces and Archimedes principle

Last Lecture we covered:

- Hydrostatic force distributions on submerged surfaces.
- Integral measures of these force distributions acting through a single point, the direction of this integral force.
- Balancing this with a moment from a hinge on a gate.
- Using free body diagrams to work out the forces on curved gates.

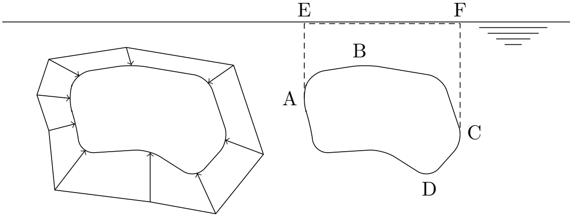
This lecture we are going to cover:

- Why submerged objects float or sink.
- Archimedes Principle.
- Touch on centre of buoyancy, centre of weight and stability

Upthrust

It is nothing to do with what is inside the object !

Upthrust arises on bodies wholly or partially submerged, because of the difference in pressure from top to bottom.



Upward force on ADC = weight of fluid above it, ie the weight of volume ADCFE.

Downward force on ABC = weight of fluid above it, ie ABCFE.

Net upthrust = weight of fluid displaced, ie in ADCB.

This is Archimedes Principle : $F_{upthrust} = \rho_{fluid} V_{object} g$

Note : Buoyancy the density of the body itself is immaterial to the amount of upthrust produced, it does of course affect the way in which the body responds to this upthrust.

Case Studies : Density of the body relative to the density of the fluid.

Floating versus Sinking

Case 1 : $\rho_{object} = \rho_{fluid}$: Body is neutrally buoyant and will sit wherever it is placed in a constant density fluid

Case 2 : $\rho_{object} < \rho_{fluid}$: Body will float so that only the submerged part of the volume contributes to the upthrust.

Case 3 : $\rho_{object} > \rho_{fluid}$: Body will sink. The speed of the motion will create an additional drag force which at equilibrium becomes its terminal speed.

$$F_{weight} = F_{upthrust} + F_{drag}$$

Summary of this Lecture/Suggested Reading

Lecture Summary

- Archimedes Principle, and that the upthrust force on a submerged body is the weight of fluid displaced
- Depending on the density of the body submerged, it will either float, be neutrally buoyant, or sink.
- If it sinks an additional drag force appears, which is proportional to the speed at which the object sinks.

Suggested Further Reading

- Crowe et al. ‘Engineering Fluid Mechanics’, 9th ed. SI version, Section 3.6-3.7
- Shrimpton, section 4.9.4

Suggested Further Study

- Complete chapter 8 questions.

Lecture 18: Dimensional analysis.

Last Lecture we covered:

- Archimedes Principle, and that the upthrust force on a submerged body is the weight of fluid displaced
- Depending on the density of the body submerged, it will either float, be neutrally buoyant, or sink.
- If it sinks an additional drag force appears, which is proportional to the speed at which the object sinks.

This lecture we are going to cover:

- Introduction to dimensional analysis.

Dimensional analysis : Dimensions.

Our fundamental principles ..

- Conservation of mass.
- Conservation of momentum - Newtons Second Law.
- Conservation of energy - First Law of Thermodynamics.

Require us to..

- define these physical quantities
- define regular amounts of them so our conservation equations are consistent.

Fundamental Dimensions

These are the basic building blocks of all other variables and are denoted by [...].
There are no fundamental reasons why we have these ones and not others.
Nevertheless they are:

- Mass $[M]$.
- Length $[L]$.
- Time $[T]$.
- Temperature $[\theta]$ (only rarely used).

Key derived dimensional quantities.

General Variable Dimensions

In general any variable may be defined in terms of dimensions $[M]^a[L]^b[T]^c$.
These are our dimensional building blocks.

Common Variables

- Momentum is mass × velocity and Velocity $U \equiv [L][T]^{-1}$.
- Acceleration is the rate of change of velocity with time or $A \equiv [L][T]^{-2}$.
- Force is mass × acceleration or $F = [M][L][T]^{-2}$.
- Stress and/or pressure is force per unit area $P \equiv [M][L]^{-1}[T]^{-2}$.
- Energy is force × distance $F \equiv [M][L]^2[T]^{-2}$.

Example

Working out the forgotten dimensions of a variable if you can remember an equation.
Work out the dimensions of pressure, use the relation $p = \rho g h$.
So $\equiv [M][L]^{-3}[L][T]^{-2}[L] \equiv [M][L]^{-1}[T]^{-2}$.

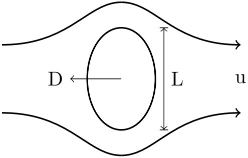
Dimensions are not Units.

Scale issues

Dimensions of a variable do not change even if the associated units do.

For example the dimensions of velocity are $U \equiv [L][T]^{-1}$.

The units used to measure velocity may be km/hour, knots, mm/s etc.



Note

Units define the *scale* of the variable.

Dimensional Homogeneity.

Total Velocity = velocity(1) + velocity(2) !

- Every term in any equation must have the same dimensions.
- If the equation is dimensionally wrong then is has no physical meaning, and must be incorrect.
- Every term in every equation must also have consistent units. All SI units.
- This is an excellent way to check your equation derivations and your exam solutions.

Example

Check the dimensions of the hydrostatic equation, $p = p_a + \rho g z$.

- $LHS = p(\frac{F}{A}) \equiv [N][L]^{-2} \equiv [M][L][T]^{-2}[L]^{-2} \equiv [M][L]^{-2}[T]^{-2}$.
- $RHS = \rho g z \equiv [M][L]^{-3}[L][T]^{-2}[L] \equiv [M][L]^{-1}[T]^{-2}$.
- $LHS = RHS$. OK
- Bonus checking : for numerical problems ensure units are consistent (kJ and J a classic error)

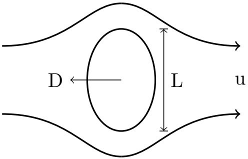
Dimensional Analysis

How to reduce how many experiments you do..

We can use dimensional analysis to predict the form of a mathematical relationship between say, things we control, and things we measure in an experiment.

We use dimensional analysis to design the types of experiments we perform.

Example : Let us say we wish to measure the drag force D exerted on an object of size L in a flow of a given fluid of velocity U.



What could/should the drag depend on ? $D = f(L, U, \rho, \mu)$.

Dimensional Analysis: example.

Non-Dimensional Functional Relationship

Let us say the Drag $D \equiv [N] \equiv [M][L][T]^{-2}$ depends on:

Length $L \equiv [L]$, Velocity $U \equiv [L][T]^{-1}$, Density $\rho \equiv [M][L]^{-3}$, Viscosity $\mu \equiv [M][L]^{-1}[T]^{-1}$.

$D = f(L, U, \rho, \mu)$.

Let us try to make the LHS of this non-dimensional - we divide the LHS by using some of our variables, in some combination to make the term have no dimensions.

$\rho L^2 U^2 \equiv [M][L]^{-3}[L]^2[L]^2[T]^{-2} = [M][L][T]^{-2}$.

So now we can re-write our dimensional functional relationship, as a non-dimensional functional relationship:

$\frac{D}{\rho L^2 U^2} = g(L, U, \rho, \mu)$.

Dimensional Analysis: example.

Defining the RHS of our relationship

Originally that Drag depends on 4 variables, and we have used 3, $\frac{D}{\rho L^2 U^2} = g(L, U, \rho, \mu)$.
Therefore the term on the right hand side must:

- Use the unused variable (viscosity).
- Be non-dimensional.
- So we have to use the other 3 variables again to make another non-dimensional quantity.
- $\rho U L$ has the same dimensions at μ , so:

$$\frac{D}{\rho L^2 U^2} = h\left(\frac{\rho U L}{\mu}\right)$$

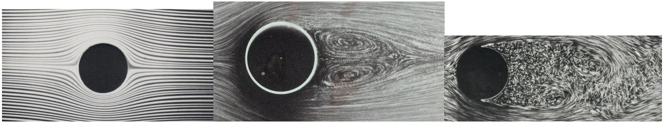
We have discovered that...

The *non-dimensional* drag depends on a collection of variables which together have no dimensions.
So we only have to plot the these on a 2D plot to know everything about this problem.
Non-dimensional quantities the KEY way we understand thermofluids.
We have discovered two, the Drag Coefficient and the Reynolds Number

Dimensional Analysis: example.

The Reynolds Number defines the *stability* of the flow.

For low Re, the flow is stable (laminar).
Beyond some critical Reynolds number the flow becomes unstable.
Eventually turbulent. Drag in laminar and turbulent flow is very very different.



Using Scale Models for Real Scale Engineering

Suppose we would like to design an airship..

It might be 10m in size, move at 1m/s and move through air.

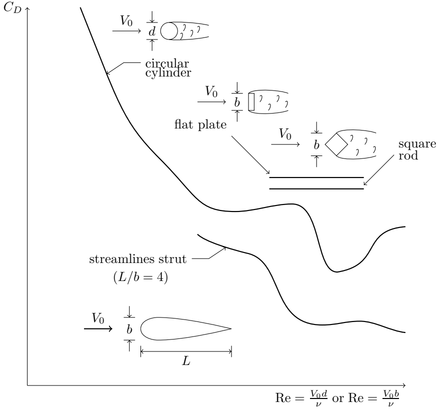
We might want to design a model 10cm in size, moving at 0.1m/s

We can match the Reynolds Numbers, eg

$$\left(\frac{\rho UL}{\mu}\right)_{airship} = \left(\frac{\rho UL}{\mu}\right)_{model}$$

We then measure the model drag, plot it non-dimensionally

This is known as Reynolds number similarity (cf geometric similarity).



Summary and Suggested Reading

Lecture Summary

- Defined what a dimension and a unit is, and how they are different.
- Introduced the concept of dimensional homogeneity in equations, consistent units
- Laid out a procedure to create sets of non-dimensional groups of variables.
- Showed these non-dimensional groups have real physical meaning via the concept of dynamic similarity.
- Pointed out we can use dimensional analysis to guide the design of experiments and undertake scaled model analysis whilst retaining a constant value of a key non-dimensional group.

Suggested Further Reading

- Crowe et al. Section 1.3, 1.4, 8.1
- Shrimpton, Section 2.4

Suggested Further Study

- Try chapter 4 questions. Worked solution on other possible repeating groups.

Lecture 19: The Buckingham Pi Theorem.

Last Lecture we covered:

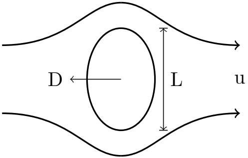
- Defined what a dimension and a unit is, and how they are different.
- Introduced the concept of dimensional homogeneity in equations, consistent units
- Laid out a procedure to create sets of non-dimensional groups of variables.
- Showed these non-dimensional groups have real physical meaning via the concept of dynamic similarity.
- Pointed out we can use dimensional analysis to guide the design of experiments and undertake scaled model analysis whilst retaining a constant value of a key non-dimensional group.

This lecture we are going to cover:

- The formal procedure for using the Buckingham Pi theorem.

Number of Groups = Number of Variables - Number of Dimensions.

Buckingham Pi Procedure (based on the first example of the previous lecture).



- Number of Variables = 4.
- Number of Dimensions = 3.
- Number of Groups = 1.

Steps to obtain the repeating group

1. Choose at least 3 variables which together contain all (3) the dimensions present.
2. Write the non-dimensional left hand side as $D\rho^a L^b U^c$, which gives:
3. $\underbrace{[M][L][T]^{-2}}_{D}\left\{\underbrace{[M][L]^{-3}}_{\rho}\right\}^a\left\{\underbrace{[L]}_{L}\right\}^b\left\{\underbrace{[L][T]^{-1}}_{U}\right\}^c$, which must be non-dimensional.
4. So compare the coefficients for the dimensions, gives 3 equations.

$[M]...1 + a = 0$
$[L]...1 - 3a + b + c = 0$
$[T]... - 2 - c = 0$
Which gives $a = -1, b = -2, c = -2$ and therefore $D\rho^{-1}L^{-2}U^{-2}$.

Repeat for each of the non-dim ratios on the right hand side.

Our remaining variable was μ

Our repeating variables are $\rho L U$, find the coefficients of:

- $\mu \rho^a L^b U^c$, e.g.
- $\underbrace{[M][L]^{-1}[T]^{-1}}_{\mu} \left\{ \underbrace{[M][L]^{-3}}_{\rho} \right\}^a \left\{ \underbrace{[L]}_{L} \right\}^b \left\{ \underbrace{[L][T]^{-1}}_{U} \right\}^c$

Compare the coefficients:

$$[M] \ldots 1 + a = 0$$

$$[L] \ldots -1 - 3a + b + c = 0$$

$$[T] \ldots -1 - c = 0$$

Leads to $a = b = c = -1$ and therefore $\mu \rho^{-1} L^{-1} U^{-1}$.

Repeating and Remaining variables.

The repeating variables in this case were $\rho L U$

They have the following properties.

- It is important that these chosen variables are not able to be expressed non-dimensionally on their own.
- They contain all the dimensions
- Same variable count as dimensions In this case it is impossible because only velocity has units of time. This fact defines your repeating variable group.

The remaining variables are each non-dimensionalised by one or more of the repeating group, and each becomes a new non-dimensional group.

General steps (problem of any size)

.

- Work out how many groups you expect from the variables and the dimensions you have in your problem.
- Find the largest number of variables that cannot be formed into a dimensionless group (usually this is equal to the number of dimensions present) - this group is your repeating variable group.
- Sequentially work out these non-dimensional groups with each remaining variable.

Original work was done by Buckingham, E, Trans ASME, 1915. He called the groups "Pi groups", hence the *Buckingham Pi* theorem.

Example: Hydraulic Jumps

Occurs when a free surface is present occasionally

Clearly for this flow there are two relevant length scales y_1 and y_2. as well as the velocity U, the acceleration g and the fluid density and dynamic viscosity.

Find the non-dimensional functional relationship between the variables, e.g. $f(y_1, U, \rho, \mu, g) = y_2$.

Number of groups, and the repeating variables

$$y_2 = f(y_1(m), U(ms^{-1}), \rho(kgm^{-3}), \mu(kgm^{-1}s^{-1}), g(ms^{-2}))$$

Number of Variables = 5

Number of Dimensions $[L], [M], [T]$ = 3

Number of extra Groups = 2 (in addition to the one with y_2)

y_1, U, ρ are the largest number of variables that cannot be formed into a group, so these are our repeating group. The other three variables are another entirely reasonable group also note.

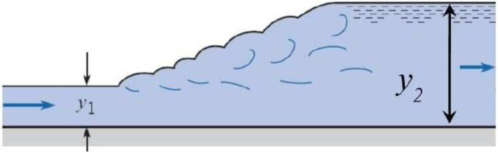
So our extra 2 Pi groups are formed from the 2 remaining variables, with the repeating group.

$$\Pi_1 = f_1(y_2, y_1, U, \rho)$$

$$\Pi_2 = f_2(g, y_1, U, \rho)$$

$$\Pi_3 = f_2(\mu, y_1, U, \rho)$$

Solution.



$$\Pi_1 = f_1(y_2, y_1, U, \rho) = y_2 y_1^a U^b \rho^c \equiv \left\{ \underbrace{[L]}_{y_2} \right\} \left\{ \underbrace{[L]}_{y_1} \right\}^a \left\{ \underbrace{[L][T]^{-1}}_{U} \right\}^b \left\{ \underbrace{[M][L]^{-3}}_{\rho} \right\}^c$$

$[M] \ldots c = 0$

$[L] \ldots 1 + a + b - 3c = 0$

$[T] \ldots - b = 0$

$c = b = 0, a = -1.$ $\Pi_1 = \left(\frac{y_2}{y_1} \right)$

Similarly: $\Pi_2 = \left(\frac{U^2}{g y_1} \right)$, $\Pi_3 = \left(\frac{\rho U y_1}{\mu} \right)$ and $\left(\frac{y_2}{y_1} \right) = f \left(\frac{U^2}{g y_1}, \frac{\rho U y_1}{\mu} \right)$

Another important non-dimensional group is the Froude Number.

Summary and Suggested Reading

Lecture Summary

- Introduced the methodology behind the Buckingham Pi Theorem :
- Work out how many groups you expect from the variables and the dimensions you have in your problem.
- Find the largest number of variables that cannot be formed into a dimensionless group (usually this is equal to the number of dimensions present) - this group is your repeating variable group.
- Sequentially work out these non-dimensional groups as shown above.
- Introduced the Froude Number

Suggested Further Reading

- Crowe et al. Section 8.2-8.9 - note provides an alternative method which some may find useful.
- Shrimpton Section 2.4

Suggested Further Study

- Complete chapter 4 questions.

Lecture 20: Flow Assumptions and Boundary Conditions

Last Lecture we covered:

- Introduced the methodology behind the Buckingham Pi Theorem.
- Work out how many groups you expect from the variables and the dimensions you have in your problem.
- Find the largest number of variables that cannot be formed into a dimensionless group (usually this is equal to the number of dimensions present) - this group is your repeating variable group.
- Sequentially work out these non-dimensional groups as shown above.
- Introduced the Froude Number

This lecture we are going to cover:

- Flow Assumptions - reduce the complexity.
- Boundary Conditions - what to assume at walls, edges of the domain.
- *How to simplify problems and solve them*.

Physical Properties.

Default Condition

In this course, all physical properties are constant and non-zero except:

Density

- When considering system thermodynamics problems (leading to heat engines)
- Some flow energy problems (lectures 26 and 27)

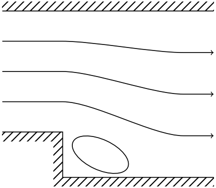
Viscosity

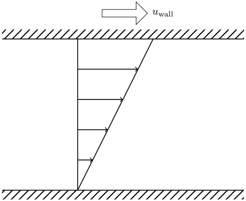
- Viscosity : Zero when the fluid is inviscid. (most of the time except Lectures 30-33).

Flow Dimensionality (key words to find in questions)

Reducing the dimensions to make it solvable

In the workbook, and in exam questions, you are required to solve problems that are solvable analytically (ie using maths). A key skill is to simply your problem to a solvable state, and an important step is to reduce the problem dimension.





'large' scales

Usually the geometry will be large in one or more directions then nothing changes in that direction.
In both cases we can safely assume that $\frac{\partial}{\partial z}(\ldots) = 0$

$U = f(x,y), V = f(x,y)$ $U = f(y)$

Flow Assumptions (key words to find in questions).

Steady Flow

If the flow is said to be steady then there is no change in the solution in time at a given spatial location. Therefore:

$\frac{\partial}{\partial t}(\ldots) = 0$ or $\frac{d}{dt}(\ldots) = 0$ or $U(x, y, z, t) = U(x, y, z)$ or $U(x, t) = U(x)$

Uniform Flow

If the flow is stated to be uniform then it does not vary in space. Therefore

$U(x, y, z) = (A, B, C)$ or $U(x) = A$

Fully Developed

If the flow is said to be fully developed then the fluid velocity and all other variables are constant in the main flow direction. Therefore, if the flow is in the x direction then

$\frac{\partial}{\partial x}(\ldots) = 0$ for all variables *except pressure*

The pressure is a special case. Here the pressure *gradient* is also constant (may be zero).

Example: if the flow is fully developed in the x direction then:

$\frac{\partial}{\partial x}(\ldots) = 0$ except $\frac{\partial p}{\partial x} = C$ where C might be 0.

Boundary Conditions: Walls.

Viscous Flow Wall Boundary Condition

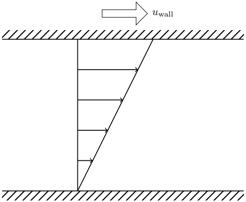
At a solid surface the no-slip condition applies if the fluid is viscous. This means the fluid, next to the wall, moves with the wall.

Fluid Tangential Velocity Assumption at the Wall

The velocity tangential to the surface, at the surface is the same as the wall velocity. For the 1-D example $U = U_{wall}, W = 0$ at $y = h$. $U = 0, W = 0$ at $y = 0$.

Fluid Normal Velocity Assumption at the Wall

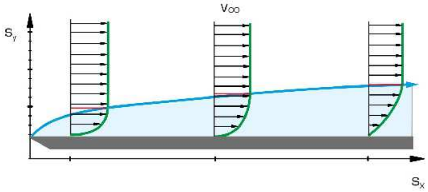
Default : For solid walls, the normal velocity is also zero. For the 1-D example $V = 0$ at $y = 0$ and $y = h$.
For porous walls, the normal velocity is defined (in the question). For a 1-D example wall $V = V_{wall}$ at $y = 0$ and $y = h$.



Lecture 20: Boundary Conditions: Boundary Layer at 'infinity'.

Boundary Layers

These are a thin layer of fluid near an objects surface that is moving through a (still) fluid. They are the origin of the viscous drag force.



precisely..

$U(y) \rightarrow U$ as $y \rightarrow \infty$.

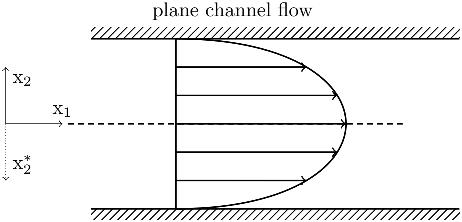
Also means that: $\frac{dU}{dy} \rightarrow 0$ as $y \rightarrow \infty$.

practically..

$U(edge) = 0.99U(\infty)$

Boundary Conditions: Symmetry Plane (x,y,z coord system).

Fully Developed, Steady Flow between two large flat plates



A symmetry plane is present if the solution can be reflected across it.

Special properties are:

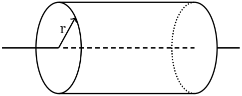
- Zero gradients normal to the boundary, at the boundary.
- Zero normal velocity at (through) the boundary.

Rarely used in part 1. Used a great deal parts 3 to 4.

Boundary Conditions: Symmetry axis (r, θ, z coord system).

Fully Developed, Steady Flow in a round pipe

Cylindrical systems $U(r, \theta, z)$ have an axis at $r = 0$.



$U_1 = U_r = 0$ at $r = 0$

$U_2 = U_\theta = 0$ at $r = 0$

$\frac{\partial U_3}{\partial r} = \frac{\partial U_z}{\partial r} = 0$ at $r = 0$

Rarely used in part 1. Used a great deal parts 3 to 4.

Examples

7. Water flows through a long pipe of diameter 10 cm. Assuming fully developed flow and that the pressure gradient along the pipe is 400 Nm^{-3}, perform an overall force balance to show that the frictional stress acting on the pipe wall is 10 Nm^{-2}. What is the velocity gradient at the wall?

Consider a large tank of fluid open to the atmosphere, connected to a pipe near its base, which branches to two more pipes each of which is open to the atmosphere (see Figure 10.9). Obtain the four equations required to solve for flow in this inviscid fluid system, defining all terms you use.

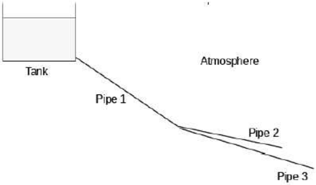


Figure 10.9: *Pipe system connected to a large water tank.*

Summary and Suggested Reading

Lecture Summary

- Fluid Property Assumptions.
- Problem Simplification (Flow Assumptions)
- Boundary Conditions (Integration Constants)
- Some Examples

Suggested Further Reading

- Shrimpton Section 2.6.4

Suggested Further Study

- Scan (do not solve) questions from Chapters 8,9,10,11,12 highlighting key words.

Lecture 21: Convective and Diffusive Transport.

Last Lecture we covered:

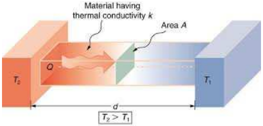
- Fluid Property Assumptions.
- Problem Simplification (Flow Assumptions)
- Boundary Conditions (Integration Constants)
- Some Examples

This lecture we are going to cover:

- The mechanism of diffusion and convection.
- Definition of convective and diffusive flux through a control volume surface.
- Balance of convection and diffusion in Fluids.

Lecture 21: Convection AND Diffusive Transport

Diffusion (of heat)



Diffusion always transports *stuff* down a gradient.

Convection (of sticks)

Convection always transports *stuff* in the flow direction

Convection and Diffusion Acting Together

Wax Diffusion, Heat Diffusion and Convection

The Convection-Diffusion of Energy is the subject of this lecture.

Mass, Momentum and Energy Reminder

Conservation of *extensive* A is...

Accumulation of A in ΔV = Net Flow of A across the surface of ΔV + Net creation rate of A in ΔV.
where..

Mass $(A = m)$	Density $(a = l)$
Momentum $(A = M_x = mU_x)$	Velocity $(a = U_x)$
Enthalpy $(A = E_h)$	Specific enthalpy $(a = e_h = C_p T)$

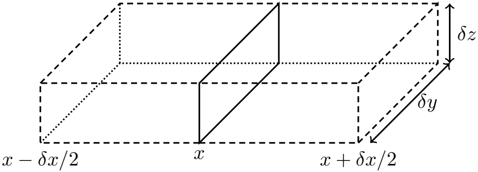
For any extensive property A, its corresponding intensive property a may be defined,

$$A = \int_V \rho a \partial V \approx \rho a \Delta V.$$

Transport of enthalpy due to Convection though a surface

Convective flow of an intensive variable (specific enthalpy)

The flow of any convected variable other than mass is the mass flow multiplied by the intensive form of that variable.

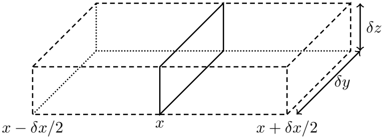


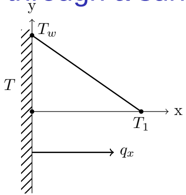
For instance, enthalpy flow (Js^{-1}) = mass flow $(kgs^{-1}) \times$ specific enthalpy (Jkg^{-1}).

Hence: Known enthalpy flow at $x = X$: $\dot{m}e_h = \rho AUC_P T = \rho UC_P T \delta y \delta z$.

We can do this for any intensive variable $a = 1, U, e_h$.

Transport of enthalpy due to Diffusion through a surface





Fourier's Law defines the thermal *flux*

Heat flows down the thermal gradient, at a rate controlled by the thermal conductivity : $q_x = -k \left. \frac{dT}{dx} \right|_X$.

Heat *flow* is flux x area

The Heat Flow (Js^{-1}) at the control volume centre, in the x-direction is: $Aq_x = q_x \delta y \delta z$.

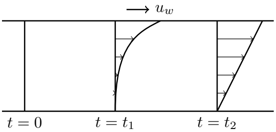
Scalar (Heat) Wall Boundary Conditions

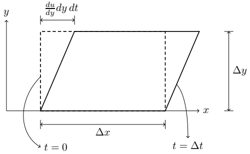
AT the wall, $q_x = -k \left. \frac{dT}{dx} \right|_{WALL}$. So $q_x = -k \frac{T_w - T_1}{\Delta x}$

Assumptions

- Constant Wall Temperature : $T_w = const$. Heat can flow in or out depending on whether the temperature of the fluid is lower or higher than the wall.
- Constant Wall Flux : $q_x = const$. The temperature at the wall is not fixed. A special case is zero flux, which defines an *adiabatic* boundary condition.

Transport of Momentum due to diffusion





Fluid is in motion - the top wall is moving and the bottom wall is not

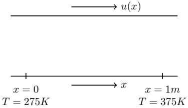
Rate of Shear Strain = $\frac{\text{Deformation in x}}{\text{length in y}} / \text{time} = \frac{\frac{dU}{dy}\delta y \delta t}{\delta y} / \delta t = \frac{dU}{dy}.$

- The force at the top wall is the shear force per unit area - the shear stress in the x-direction.
- It is proportional to the velocity gradient in the y-direction through the viscosity coefficient : $\tau = \mu \frac{dU}{dy}$

Notes

- Sometimes you might see it written τ_{xy}. This means "shear stress in the x-direction acting on a area normal to the y-direction".
- μ is known as the "dynamic viscosity", sometimes you might see $\nu = \frac{\mu}{\rho}$, which is known as the kinematic viscosity.
- A *linear* relation between the stress and the rate of strain means the fluid is Newtonian

Control Volume Analysis of 1-D enthalpy Convection-Diffusion



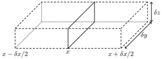
Test Problem Specification

- We have a long channel in the x-direction. The walls large in the y and the z-direction.
- Assumptions : 1-D velocity, x-direction only $U = Const, V = W = 0$ Steady conditions, the fluid is incompressible.
- We are interested in a section of this channel from $x = 0$ to $x = 1m$.
- Boundary Conditions $x = 0$, $T = 273K$ and at $x = 1$, $T = 373K$.
- The information to be transported is thermal energy.

Why is this interesting ?

- Convection is trying to "blow" "cold" thermal energy at 273 K from left to right.
- Diffusion is trying to spread "hot" thermal energy from right to left.
- This suggests the temperature profile over the range $1 > x > 0$ will be dependent on the relative strength of convection and diffusion.

Zeroing in on the control volume, centred on $x = X$



Control Volume Analysis requires conservation over the surface.

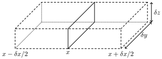
- We assume we know *everything* about the fluid and the flow at X, $U|_{x=X}$, $T|_{x=X}$, $\frac{dT}{dx}$
- No y or z flux of convection or diffusion.
- We need to employ a Taylor Series to estimate information at the control volume faces.
- Upstream Face : $T|_{X-\delta x/2} = T|_X - \left.\frac{dT}{dx}\right|_X \frac{\delta x}{2}$
- Downstream Face : $T|_{X+\delta x/2} = T|_X + \left.\frac{dT}{dx}\right|_X \frac{\delta x}{2}$.

We are going to apply a general conservation law in our small volume : The principle is, for some property A, that this property is conserved. That is...

Accumulation of A in ΔV = Net Flow of A across the surface of ΔV + Net creation rate of A in ΔV.

Here : 0 = Net Flow of A across the surface of ΔV.

Conservation of enthalpy due to convection flux through the CV surfaces



Convective flow of energy

enthalpy flow (Js^{-1}) = mass flow $(kgs^{-1}) \times$ specific enthalpy (Jkg^{-1}).
enthalpy flow at $x = X$: $C_X = \dot{m}e_h = \rho AUC_P T = \rho UC_P T\delta y\delta z$.

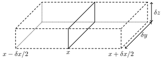
Estimating the convective enthalpy flow at the CV surfaces

- T varies, U, C_p, ρ are constant. Use Taylor series expansions from X to $X + \delta x/2, X - \delta x/2$
- Downstream surface : $T|_{X+\delta x/2} = T|_X + \left.\frac{dT}{dx}\right|_X \frac{\delta x}{2}$
- Upstream surface : $T|_{X-\delta x/2} = T|_X - \left.\frac{dT}{dx}\right|_X \frac{\delta x}{2}$.

CV Energy conservation, Convection only, Net enthalpy flow

- out - in =0, $C_U - C_D = 0$, so.. $\left.\frac{dT}{dx}\right|_X UC_P\rho\delta x\delta y\delta z = 0$ or $\rho C_P U\frac{dT}{dx} = 0$.

Conservation of enthalpy due to diffusion flux through the CV surfaces



Diffusive flow of energy

- At X, we know the thermal flux $(J/(sm^2))$: $q_x = -k \left.\frac{dT}{dx}\right|_X$.
- The area of the control volume surfaces normal to the x_1-axis is $\delta y \delta z$.
- Diffusion energy flow, at the control volume centre, in the x-direction is $D_X = q_x \delta y \delta z$.

Estimating the diffusive enthalpy flow at the CV surfaces

- T varies, U, C_p, ρ are constant. Use Taylor series expansions on q_x from X to $X + \delta x/2, X - \delta x/2$
- Downstream surface : $q_{X+\frac{\delta x}{2}} = q_x + \frac{\delta x}{2}\frac{d(q_X)}{dx}$.
- Upstream surface : $q_{X-\frac{\delta x}{2}} = q_x - \frac{\delta x}{2}\frac{d(q_X)}{dx}$.

CV Energy conservation, Diffusion only, Net enthalpy flow

- out - in =0, $D_U - D_D = 0$, so.. $+\delta x \delta y \delta z \left(\frac{d(q_X)}{dx}\right) = -\delta x \delta y \delta z k \frac{d^2 T}{dx^2}$

Convection AND Diffusion flow Energy Conservation

Recall, the basis for any conservation law is:

Accumulation of A in ΔV = Net Flow of A across the surface of ΔV + Net creation rate of A in ΔV.

For our example 1^{st} term of the LHS = Net creation rate = 0

However we have two methods of transporting information across the surface: Convection and diffusion.

Therefore for this problem the energy conservation law is:

$$\rho C_P U \frac{dT}{dx} \delta x \delta y \delta z - \delta x \delta y \delta z k \frac{d^2T}{dx^2} = 0.$$

Or $U \frac{dT}{dx} = \frac{k}{\rho C_p} \frac{d^2T}{dx^2}$.

We will examine this equation in more detail next lecture.

Lecture 21: Summary and Suggested Reading

Lecture Summary

- The mechanism of diffusion and convection.
- Definition of convective and diffusive flux through a control volume.
- Fourier's Law (for diffusive heat flux).
- Newtonian Stress-Rate of Strain relation
- Balance of convection and diffusion.
- http://www.telegraph.co.uk/news/newstopics/howaboutthat/11823069/Why-Christopher-Robin-lost-at-Poohsticks-his-stick-was-too-thin.html
- (complete nonsense !)

Suggested Further Reading

- Section 2.4 Crowe et al.
- Shrimpton, Section 2.6.1 - 2.6.5

Lecture 22: Non-dimensional analysis of Convection-Diffusion Problems.

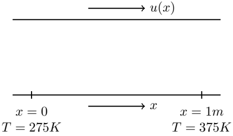
Last Lecture we covered:

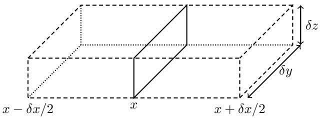
- The mechanism of diffusion and convection.
- Definition of convective and diffusive flux through a control volume.
- Fourier's Law (for diffusive heat flux).
- Newtonian Stress-Rate of Strain relation
- Balance of convection and diffusion.

This lecture we are going to cover:

- Analysis of steady 1-D convection diffusion problems.
- Non-dimensional analysis of the equation - Prantl, Reynolds and Peclet Number.
- Reynolds Number and Turbulence.

Our Enthalpy Convection-Diffusion equation and the problem.





For this problem the energy conservation law is

$$U \frac{dT}{dx} = \frac{k}{\rho C_p} \frac{d^2T}{dx^2}$$

The coefficients of the 2^{nd} order term are all material constants - assume:

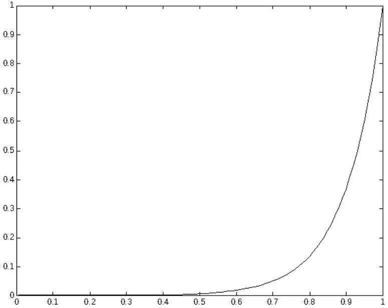
$C = \frac{k}{\rho C_p}$, therefore

$U \frac{dT}{dx} = C \frac{d^2T}{dx^2}$.

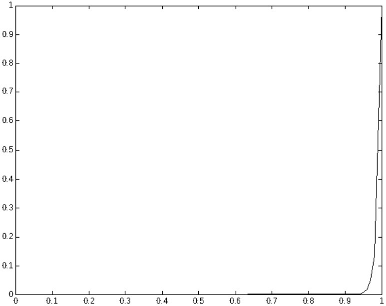
- X varies from 0 to 1.
- T at $x = 0 == 0$.
- T at $x = 1 == 1$.
- U and C are constants. For C best to think of only k can change).

We can derive the exact solution (not part of this module) to show how the values U,C affect the $T(x)$ profile.

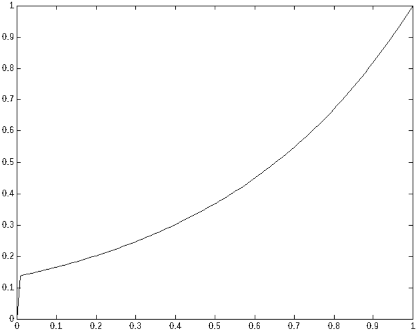
Example Results : T(1)=1.T(0)=0.,U=10,C=1.



Example Results : T(1)=1.T(0)=0.,U=100,C=1.



Example Results : T(1)=1.T(0)=0.,U=10,C=5.



Extracting Scales from the Fluid Properties.

Extracting Scales from Equations to understand them

In what follows fluid physical properties (e.g. k) are normalised (e.g. k^*) by using a reference value k_o.
This is constant throughout the domain (e.g. k_o).
The same applies to spatial and temporal scales, and also differential quantities.

Normalised Fluid Physical Properties

Physical property	**SI units**	**Symbol**	**Normalised parameter**
Thermal conductivity	$Wm^{-1}K^{-1}$	k	$k^* = \frac{k}{k_o}$
Specific heat	$Jkg^{-1}K^{-1}$	C_p	$C_p^* = \frac{C_p}{C_{po}}$
Density	kgm^{-3}	ρ	$\rho^* = \frac{\rho}{\rho_o}$
Viscosity	Nsm^{-2}	μ	$\mu^* = \frac{\mu}{\mu_o}$
Mass gas constant	J/kgK	R	$R^* = \frac{R}{R_o}$

Example

Imagine we have a variable viscosity problem, and the viscosity ranges from $0.5E-5$to$3.2E-5Ns/m^2$.
We take $\mu_o = 10^{-5}Ns/m^2$, and so $\mu^* = 0.5to3.2$, such that $\mu = \mu_o\mu^*$.
Note that the reference holds the dimensions, and the variable quantity is dimensionless.

Extracting Scales from the Geometry and Variable of Interest

Geometry/Flow property	SI units	Symbol	Normalised parameter
Length	m	x_i	$x_i^* = \frac{x_i}{x_o}$
Velocity	ms^{-1}	U_i	$U_i^* = \frac{U_i}{U_o}$
Temperature	K	T	$T^* = \frac{T}{T_o}$
Time	S	t	$t^* = \frac{tu_o}{x_o}$

These choices are problem dependent. Here, the problem is steady, so no inherent timescale of the problem.

Scaled Differential operations.

Differential operators must also be non-dimensionalised,

$$t^* = t\frac{U_o}{x_o},\ \frac{dt^*}{dt} = \frac{U_o}{x_o},\ \frac{d}{dt} = \frac{U_o}{x_o}\frac{d}{dt^*}$$

$$x^* = \frac{x}{x_o},\ \frac{dx^*}{dx} = \frac{1}{x_o},\ \frac{d}{dx} = \frac{1}{x_o}\frac{d}{dx^*}$$

The idea behind this is that the magnitude of the variable is held in the dimensional term (U_o say) and the non-dimensional term (U^* say) is of order 0.

For instance a trivial examples is if $U = 10ms^{-1}$, $U_o = 10ms^{-1}$ and $U^* = 1$

Or, if the flow varies between $100ms^{-1}$ and $130ms^{-1}$, $U_o = 100ms^{-1}$ and $1.3 > U^* > 1$

The real power of the technique however is when groups of these dimensional variables are made non-dimensional, these groups tell you how the equations (i.e. the physics) behave.

This is a much more intuitive way to define non-dimensional numbers, *understand* what they mean.

Non-dimensional form of the Convection-Diffusion Equation.

Non-dimension

$$U\frac{dT}{dx} = C\frac{d^2T}{dx^2}$$

$$\left[\frac{U_oT_o}{x_o}\right]U^*\frac{dT^*}{dx^*} = \left[\frac{C_oT_o}{x_o^2}\right]C^*\frac{d^2T^*}{dx^{*2}}$$

$$U^*\frac{dT^*}{dx^*} = \left[\frac{C_o}{U_ox_o}\right]C^*\frac{d^2T^*}{dx^{*2}}$$

$$U^*\frac{dT^*}{dx^*} = \frac{1}{Pe_o}\frac{d^2T^*}{dx^{*2}}$$

Note: $\frac{1}{Pe_o} = \left[\frac{C_o}{U_ox_o}\right] = \left[\frac{k_o}{\rho_oC_{po}U_ox_o}\right] = \left[\frac{\mu_o}{U_ox_o\rho_o}\right]\left[\frac{k_o}{\mu_oC_{po}}\right] = \frac{1}{Re_oPr_o}$

Dimensionless Numbers.

$\left[\frac{\rho_o U_o x_o}{\mu_o}\right]$	$\left[\frac{\text{convective flux}}{\text{viscous flux}}\right]$	Reynolds	Re_0
$\left[\frac{C_{po}\mu_o}{k_o}\right]$	$\left[\frac{\text{viscous flux}}{\text{thermal flux}}\right]$	Prandtl	Pr_o
$\left[\frac{g_o x_o}{U_o^2}\right]$	$\left[\frac{\text{body force}}{\text{inertia force}}\right]$	(Square of the inverse) Froude	Fr_o
$\left[\frac{U_p^2}{\gamma_o R_o T_o}\right]$	$\left[\frac{\text{flow speed}}{\text{wave speed}}\right]$	(Square of) Mach	Ma_o

Reynolds number and the scale range/strength of the non-linearity.

Non-dim steady constant density mass, momentum:

$$\frac{\partial U_i^*}{\partial x_i^*} = 0$$

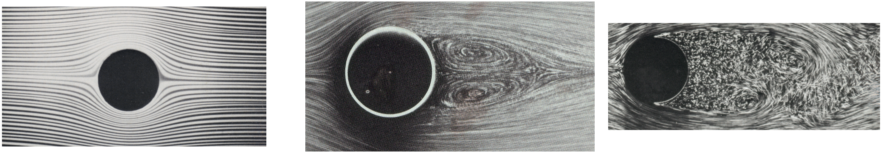
$$\frac{\partial}{\partial x_j^*}(U_i^* U_j^*) = -\frac{\partial p^*}{\partial x_i^*} + \frac{1}{Re_0}\frac{\partial \tau_{ij}^*}{\partial x_j^*}$$

Reynolds number characterises the relative importance of nonlinear-linear effects.

These are the key equations of fluid mechanics - tensor forms *NOT* part of this module.

Reynolds number and the scale range/strength of the non-linearity.

Reynolds Number Flow Dependence



- The non-dim numbers tell you real things about the physics without solving any equations.
- The conceptual leap to make in this lecture is you don’t need the equation.
- If you have the scales of the problem, and you understand what non-dim numbers mean...
- Then you know what the fluid mechanics are like in your problem.

Limiting Forms of the Navier-Stokes equations.

Full N-S Equation, assuming steady flow and no body forces

$\frac{\partial}{\partial x_j^*}(U_i^* U_j^*) = -\frac{\partial p^*}{\partial x_j^*} + \frac{1}{Re_0}\frac{\partial \tau_{ij}^*}{\partial x_j^*}$ Inertial, pressure force and viscous force present.

Euler Equations - negligible viscous forces : $Re >> 1$

$$\frac{\partial}{\partial x_j^*}(U_i^* U_j^*) = -\frac{\partial p^*}{\partial x_j^*}$$

Reasonably accurate far away from walls for air. (Studied in Section 3). We use the integral form of this. Integrate with volume to sort of arrive at.

$$M_{x,out} - M_{x,in} = \Delta PA$$

Stokes equations - inertial forces negligible : $Re << 1$

$$0 = \frac{1}{Re_0}\frac{\partial \tau_{ij}^*}{\partial x_j^*}$$

Typical applications include lubrication flows, highly viscous flows. (Studied in Section 4). We actually solve the 1-D form.

$$\frac{d^2U}{dy^2} = 0$$

But sir, why are you hurting my brain...

Both of these forms are considerably easier to solve than the full form.
None of these you will be asked to derive or solve.
Purely to demonstrate Reynolds Number dependence, and how this module is organised.

Summary and Suggested Reading.

Lecture Summary

- Analysis of steady 1-D convection diffusion problems.
- Non-dimensional analysis of the equation - Prantl, Reynolds and Peclet Number.
- Reynolds Number and turbulence.
- Navier-Stokes Equation and its simpler forms.

Suggested Further Reading

- Shrimpton, Section 2.5, Section 2.6.5. section 5.1-5.2

Suggested Further Study

- Try 4.15

Lecture 23: Flow Visualization Methods

Last Lecture we covered:

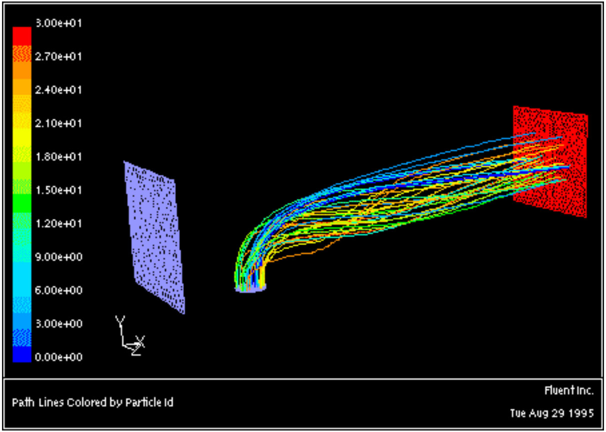
- Analysis of steady 1-D convection diffusion problems.
- Non-dimensional analysis of the equation - Prantl, Reynolds and Peclet Number.
- Reynolds Number and turbulence.
- Navier-Stokes Equation and its simpler forms.

This lecture we are going to cover:

- Describe some ways to visualize and describe the motion of fluid.
- Define the difference between the *flux* and *flow* rate.

Pathlines : A single Particle Trajectory History

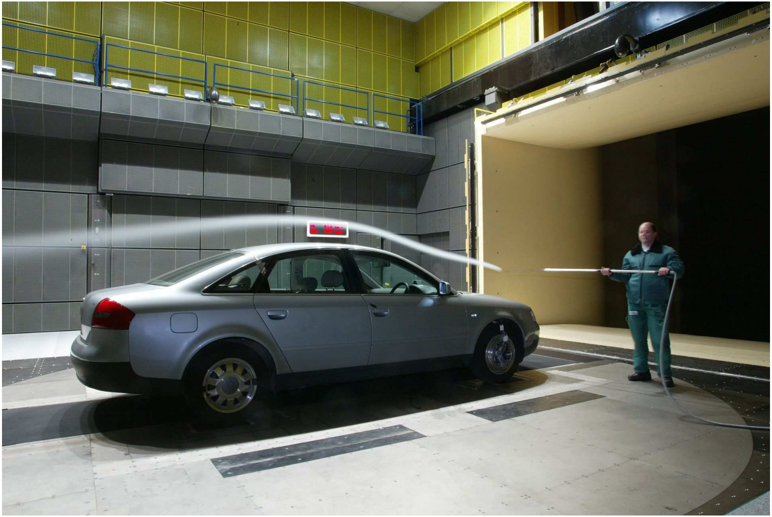
Pathline Example



Pathlines: The ’path’ traced out by a single particle released from a point over at a certain time.

Streaklines : Where a set of particles are released over time from a point

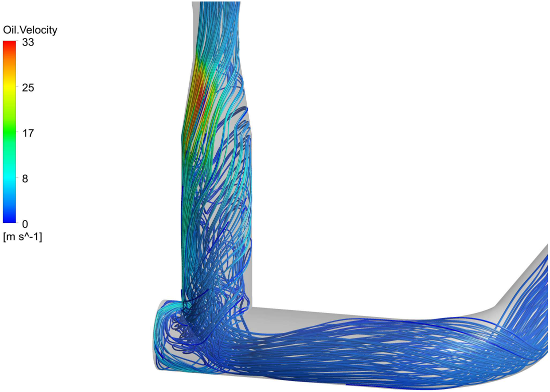
Streakline Example



Streaklines: The endpoints of many particles released from the same point over a period of time.

Streamlines : Lines connecting tangential velocity points

They are lines that are parallel to the velocity vector field

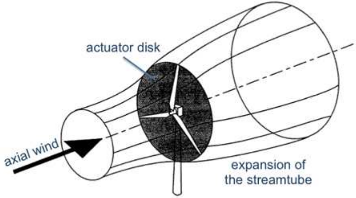


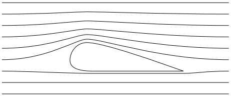
In 2D: $\frac{dx}{dy} = \frac{U}{V}$

- at one instance of time they are calculated throughout the fluid.
- So they only have physical meaning in steady flows.
- Streamlines can not cross each other. If they were to cross this would indicate two different velocities at the same point in space and time.

Collections of 'Streamlines' give you 'Streamtubes'

Streamtubes are virtual 'pipes' of fluid





A constant amount of mass flows through each tube, $\dot{m} = \rho A U_n$. This is the mass flow.

The mass flux is the flow rate per unit area. However many people use 'a flux' and 'a flow' interchangeably.

Streamtubes: Give you all the information about the fluid that streamlines do, plus an idea of the flow speed change.

Example.

For the velocity field defined by $\overrightarrow{U} = (B + Ay^2)\overrightarrow{e_1}$

(A, $B > 0$ and constant).

- Sketch the Streamlines for the following flow field on a $x - y$ axis.
- Sketch the Velocity profile on a U_1-y axis.

Summary and Suggested Reading

Lecture Summary

- A pathline is a ’path’ traced out by a single particle released from a point over a period of time.
- A streakline is the endpoints of many particles released from the same point over a period of time.
- A streamline is a line that represents an instantaneous tangent to the flow direction.
- Streamlines, Streaklines and Pathlines are only equivalent when the flow is steady.
- A stream tube is a collection of streamlines that enclose an area into which a fluid flows.
- The distribution of streamlines can tell you where the flow speeds up and where the flow slows down.

Suggested Further Reading

- Crowe et al. ’Engineering Fluid Mechanics’, 9^{th} ed. SI version, Section 4.1.
- Shrimpton, Section 4.1 (good example in Figure 4.1).

Suggested Further Study

- Try 9.11

Lecture 24: Conservation of Mechanical Force and Energy: The Euler and Bernoulli Equation.

Last Lecture we covered:

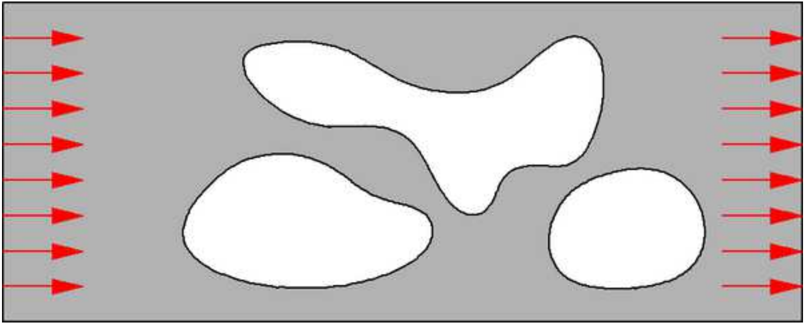
- A pathline is a ’path’ traced out by a single particle released from a point over a period of time.
- A streakline is the endpoints of many particles released from the same point over a period of time.
- A streamline is a line that represents an instantaneous tangent to the flow direction.
- Streamlines, Streaklines and Pathlines are only equivalent when the flow is steady.
- A stream tube is a collection of streamlines that enclose an area into which a fluid flows.
- The distribution of streamlines can tell you where the flow speeds up and where the flow slows down.

This lecture we are going to cover:

- Consider a force balances in a stream tube: The Euler Equation.
- Integrate along the stream tube direction to obtain an energy balance equation: the Bernoulli equation.
- Discuss the characteristics and assumptions used in creating the Bernoulli equation.

A simple example..

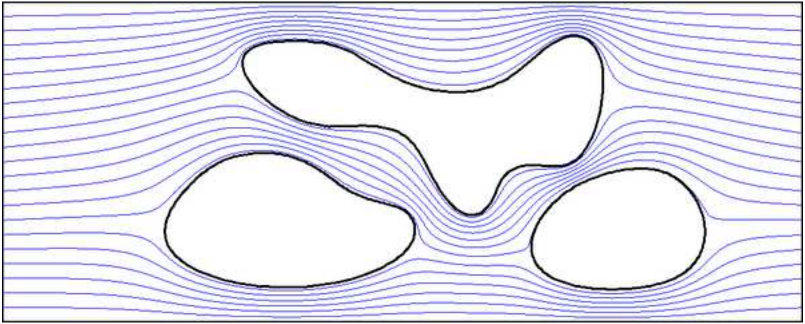
Force and Energy Conservation along a Streamline



- Consider a box of *incompressible* and *inviscid* fluid.
- There are some objects in the box, and flow is maintained at the inlet and the outlet of the box.
- Obviously the flow is going to have to accelerate in some regions, and decelerate in others

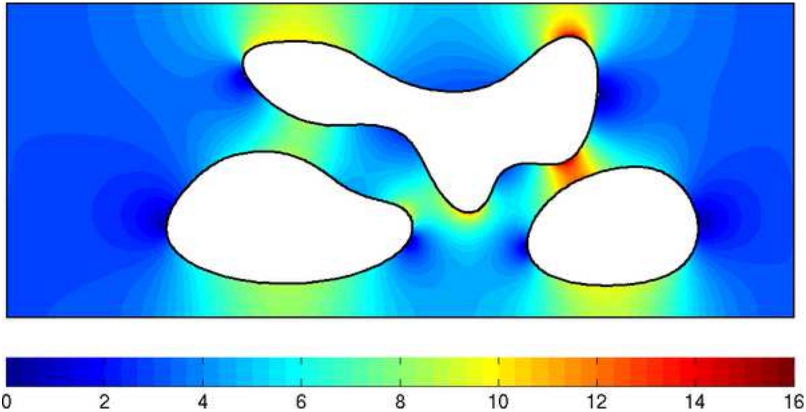
A simple example: Streamline Plot.

Flow Pattern



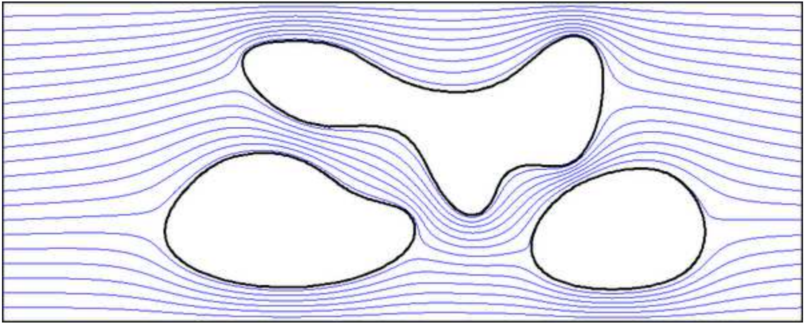
- Let us say we managed to work out how to solve the flowfield.
- So, we can plot streamlines - and this shows us where the flow speed up and slows down.
- Because the flow speeds up and slows down, there must be some force acting on the fluid volume

A simple example: Velocity magnitude plot.



Here we can see where the flow speeds up and slows down - and this must be balanced by a force related to the pressure field and the [constant] gravity force - the fluid is inviscid, so there are no shear forces.

Force Balance along a stream tube.



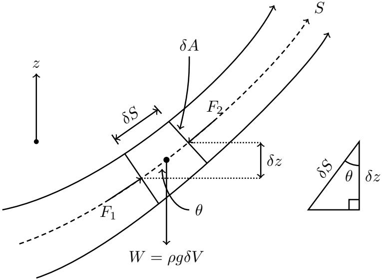
We are going to take one small section of one of these stream tubes and apply a force balance. Acceleration is a force (force per unit mass) so in words our force balance on a fluid element along the stream tube is:

'(mass $\times$ acceleration) = (pressure change $\times$ area) + (mass $\times$ gravity)'

Force components on a fluid element in a Streamtube.

Deriving the acceleration, pressure and weight terms : Force balance on S..

- The Coordinate direction along the streamline is S.
- Acceleration is $\frac{dU_S}{dt} = \frac{dU_S}{ds}\frac{ds}{dt} = U_S\frac{dU_S}{ds}$.
- The volume element has length δS and area δA hence volume $\delta V = \delta S \delta A$.
- The weight of the fluid (N) is $mg = \rho \delta V g$ acting in the Z direction, and $\rho \delta V g cos\theta$ acting in the S direction.
- The pressure force at $S = 0$ (F_1) is $p\delta A$
- The pressure force at $S = \delta S$ (F_2) is expanded as a Taylor series and is $\left(p + \frac{dp}{ds}\delta S\right)\delta A$.



So...

$$\rho \delta S \delta A U_S \frac{dU_s}{ds} = p\delta A - \left(p + \frac{dp}{ds}\delta S\right)\delta A - \rho \delta V g cos\theta$$

Simplifications, and conversion to a mechanical energy balance (Bernoulli Equation).

Starting from $\rho \delta S \delta A U_S \frac{dU_s}{ds} = p\delta A - \left(p + \frac{dp}{ds}\delta S\right)\delta A - \rho \delta V g cos\theta$

Force Balance

- Divide by $\delta S \delta A$, (2) $cos\theta = \delta z / \delta S$, Take the limit as $\delta S \rightarrow 0$, $\delta z \rightarrow 0$.
- We get a force balance $\rho U_S \frac{dU_S}{ds} + \frac{dp}{ds} + \rho g \frac{dz}{ds} = 0$.
- This is the Euler Equation. (Inviscid force balance along a streamtube)
- The total force change is zero along a stream tube. It can transfer between forms.

Energy Balance

- Integrate w.r.t. S to get a mech energy balance (Energy = Force × distance)
- $\frac{\rho U_S^2}{2} + p + \rho g z =$ Const.
- This is the Bernoulli equation. (Mechanical Energy Balance along a streamtube).
- The total mechanical energy is constant along a stream tube. It can change forms.

Assumptions inherent in deriving the Bernoulli Equation.

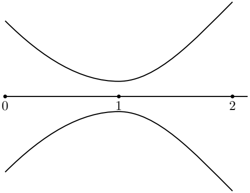
Remember, we have made several implicit assumptions by using a streamline as a basis:

- The flow must be steady - otherwise streamlines have no meaning.
- There are no viscous forces
- We have also assumed that the fluid is incompressible, but that is rather subtle.

It is important to remember these assumptions and not use the Bernoulli Equation incorrectly.

Cavitation: An example of the inviscid Bernoulli Equation.

Consider a flow through a narrowing channel



The Bernoulli equation is:

$$p_0 = \frac{\rho U_1^2}{2} + p_1 = \frac{\rho U_2^2}{2} + p_2.$$

As the flow velocity increases, the pressure decreases - for a given stagnation pressure a maximum velocity exists. For large pressures, several things can happen:

1. For compressible fluids the flow velocity exceeds the speed of sound, and the flow becomes sonic (not in this module).
2. For incompressible fluids, the flow may cavitate due to local pressure reducing to the vapour pressure (the liquid locally boils).

Boiling may be approached by either increasing the temperature (increasing the vapour pressure in the liquid), decreasing the pressure above it, or both (boiling a kettle on top of a high mountain).

Cavitation due to flow is characterised by a Cavitation Number, $Ca = \frac{p_{local} - p_\nu}{\frac{1}{2}\rho U^2}$.

Summary and Suggested Reading

Lecture Summary

- Using a stream tube as a basis, derived a force balance along the stream tube. Force is conserved along a streamline.
- We integrated this force balance to produce an energy balance equation, known as the Bernoulli equation.
- We showed that the total energy, the sum of kinetic, potential and pressure energy components is constant along a streamline.
- Because the Bernoulli equation uses a streamtube as a basis, it is only valid in steady flows. Also no viscous forces must be present and the relation presented here assumes an incompressible fluid.
- Cavitation can occur due in incompressible fluids to localised boiling when the flow velocity increases beyond the vapour pressure.

Suggested Further Reading

- Crowe et al., 9^{th} ed. SI version, Section 4.1-4.5, 7.5.
- Shrimpton, Section 4.2-4.3.

Suggested Further Study

- Try Chapter 9 questions.

Lecture 25: Momentum Conservation.

Last Lecture we covered:

- Using a stream tube as a basis, derived a force balance. Force is conserved along a streamline.
- We integrated this force balance to produce an energy balance equation, known as the Bernoulli equation.
- We showed that the total energy, the sum of kinetic, potential and pressure energy components is constant along a streamline.
- Because the Bernoulli equation uses a streamtube as a basis, it is only valid in steady flows of an inviscid fluid.

This lecture we are going to cover:

- Remind what the Force-Momentum Equation does.
- Why it is not the Euler Equation.
- How to use this, with the Bernoulli Equation.
- Examples.

Mass, Velocity, Momentum, Force and Acceleration.

Scalar and Vector Quantities

- Mass (extensive) and its intensive quantity (density - mass per unit volume) are scalars.
- Momentum (extensive) and its intensive quantity (velocity - momentum per unit mass) are vectors.
- Force (extensive) and its intensive quantity (acceleration - force per unit mass) are all vectors.

So when we write a conservation equation for mass (or density) we write one equation. When we write a conservation equation for momentum (or velocity) we write three equations, one for each direction.

- Momentum in the x-direction is $mU_x = M_x$, the momentum vector is $m\overrightarrow{U} = \overrightarrow{M}$.
- Rate of change of momentum is $\frac{d}{dt}(mU_x) = F_x = ma_x$.
- So Newton's Second Law is a rate of momentum change equation.

Before we get onto deriving a momentum conservation equation - remind ourselves of mass conservation.

To find the mass flow through a surface element.

What we want to do is find out how much of $\overrightarrow{U}$ is going in the $\overrightarrow{n}$ direction.

From simple trigonometry normal component $|\overrightarrow{U}|cos\theta$

General dot product definition:

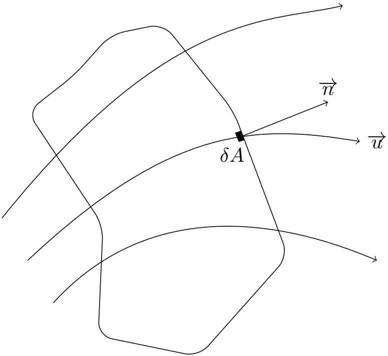
$\overrightarrow{a} \bullet \overrightarrow{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\overrightarrow{a}||\overrightarrow{b}|cos\theta$

In this case: $\overrightarrow{U} \bullet \overrightarrow{n} = |\overrightarrow{U}|cos\theta$

So the mass flow through the small element is $\delta\dot{m} = \rho(\overrightarrow{U} \bullet \overrightarrow{n})\delta A$

All elements, and mass conservation over the entire surface (volume)
$\dot{m} = \int \delta\dot{m} = \int \rho(\overrightarrow{U} \bullet \overrightarrow{n})dA$.

For incompressible fluids the net mass flow must be zero, i.e. $\int \rho(\overrightarrow{U} \bullet \overrightarrow{n})dA = 0$.



Eulerian Conservation of Momentum

We already have a Lagrangian Version..

- $\overrightarrow{F} = m\overrightarrow{a}$
- We can *automatically* get an Eulerian Version using the Reynolds Transport Theorem
- $\overrightarrow{F} = \frac{\partial}{\partial t}\int \rho \overrightarrow{U} dV + \int \rho \overrightarrow{U}(\overrightarrow{U} \bullet \overrightarrow{n})dA.$

Reynolds Transport Theorem is extremely powerful because

- We can use any variable (we will use energy later)
- ....but it does not really help us *understand*

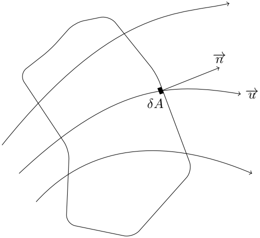
The velocity field 'carries' mass/mom information....

Flow field 'carries' density

Density = mass per unit volume = specific mass The mass flow through the small element is:

$$\delta \dot{m} = \rho(\overrightarrow{U} \bullet \overrightarrow{n})\delta A$$

Velocity field *transporting* density (ρ) information in/out of each surface element of the entire volume.



Flow field 'carries' velocity

Momentum flow in the x-direction through the element would be:

$$\delta \dot{M}_x = \rho U_x(\overrightarrow{U} \bullet \overrightarrow{n})\delta A$$

$$[kgm^{-3}.m^2 s^{-2}.m^2 = kg.ms^{-2} = N]$$

Recall : Mass and Momentum are our *extensive* variables, 1 and $\overrightarrow{U}$ the *intensive* ones.

Conservation of Momentum Equation.

Conservation of x-momentum is...

- 'Rate of accumulation of x-momentum in the volume + net rate of x-momentum leaving the volume = x-force applied to the volume'
- $\frac{\delta}{\delta t}(\int \rho U_x \delta V) + \int \rho U_x(\overrightarrow{U} \bullet \overrightarrow{n})\delta A = \sum F_x$
- Does not assume a streamline : the Euler equation is a special case of the general Force-momentum problem.

Forces

Pressure qradient, gravity (acceleration), viscous forces, exotics.

Simplifications, and where we deal with these in the module

- We will only consider steady flows in this module : $\frac{\delta}{\delta t}(\int \rho U_x \delta V) = 0$
- For inviscid flows with constant flow over inlet/outlet $\dot{M}_x = \dot{m} U_x$ and so,
- $\sum_{out} \dot{M}_x - \sum_{in} \dot{M}_x = \sum F_x$. $Re >> 1$
- one inlet+outlet $\dot{m}(U_{out,x} - U_{in,x}) = \sum F_x$
- no flow (statics !) $\sum F_x = 0$. Pressure and gravity forces.
- viscous problems : LHS=0, $\sum F_x = 0$, $Re << 1$. Viscous forces only.

Summary and Suggested Reading

Lecture Summary

- Reminded ourselves that mass and density are scalar extensive and intensive quantities.
- Invoked Newton's 2nd Law and unsteady conditions to define a general momentum conservation law.
- Simple Forms of the Momentum Equation

Suggested Further Reading

- Crowe et al. 'Engineering Fluid Mechanics', 9^{th} ed. SI version, Section 6.1-6.4, 6.6.
- Shrimpton, Section 2.2.9-2.2.20, 2.3, 4.8

Suggested Further Study

- Try chapter 10 questions.

Lecture 26: Actuator Disk theory.

Last Lecture we covered:

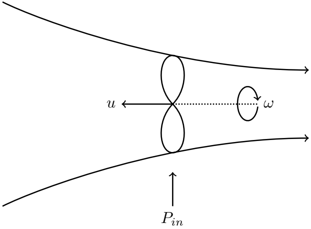
- Reminded ourselves that mass and density are scalar extensive and intensive quantities.
- Invoked Newton's 2nd Law and unsteady conditions to define a general momentum conservation law.
- Simple Forms of the Momentum Equation

This lecture we are going to cover:

- The careful combination of momentum and mechanical energy conservation.
- Propeller performance, through use of:
- Actuator disk theory.

Lecture 26: Propeller versus Turbines

Propellers: Move through the fluid



- u is the speed of the object 'speed of advance'.
- P_{in} is the power input to drive the propeller.

Turbines: Fluid moves through the turbine

- V is the speed of the oncoming fluid.
- P_{out} is the power extracted from the fluid motion.

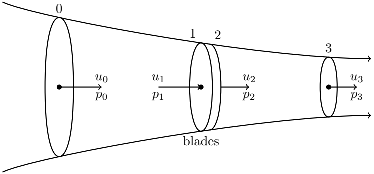
Propeller : Stationary frame

To analyse the problem, in the propeller case, we should change the frame of reference, $U = V - u$

Understanding The Propeller/Turbine

The Propeller/Turbine - as a 'black box'

- It is a *thin* disk of diameter D.
- Assume a 1-D, steady flow.
- Flow through the blades is viscous and complex.
- Causes a pressure difference.



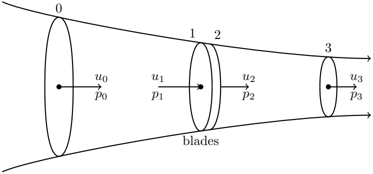
Considering the disk - the fluid state at stations 1 and 2

- $A = \frac{\pi D^2}{4}$.
- Mass conservation: $\dot{m} = \rho A U_1 = \rho A U_2 = const$. Therefore, $U_1 = U_2$.
- Momentum Conservation $\dot{m}(U_2 - U_1) = 0$. Therefore $(p_1 - p_2)A + F_x = 0$.
- Pressure difference must exist - it generates the Force (thrust).
- Bernoulli is violated, because of viscous losses due to turbulence.

Understanding the Far Field

We *can* apply Bernoulli

- Can apply Bernoulli from $0 \rightarrow 1$.
- Can apply Bernoulli from $2 \rightarrow 3$.
- Assume Stations 0 and 2 are 'far' upstream and downstream.
- We can assume $p_0 = p_3 = p_{atm}$.
- We know that $U_1 = U_2$.



Bernoulli between :

- $0 \rightarrow 1: \frac{\rho U_0^2}{2} + p_{atm} = \frac{\rho U_1^2}{2} + p_1$.
- $2 \rightarrow 3: \frac{\rho U_2^2}{2} + p_2 = \frac{\rho U_3^2}{2} + p_{atm}$.

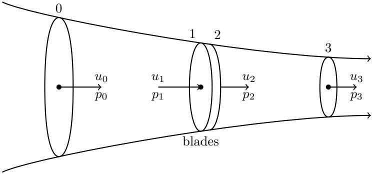
Adding these equations gives: $\frac{\rho U_2^2}{2} + p_2 + \frac{\rho U_0^2}{2} + p_{atm} = \frac{\rho U_3^2}{2} + p_{atm} + \frac{\rho U_1^2}{2} + p_1$.
Which gives, $p_2 - p_1 = \frac{1}{2}\rho(U_3^2 - U_0^2)$.

Force is

$F = (p_2 - p_1)A = \frac{\pi D^2}{8}\rho(U_3^2 - U_0^2)$.

What Actuator Disk Theory tells us

Disk theory predicts that:



- $F \propto D^2$.
- $F \propto (U_3^2 - U_0^2)$.

Now consider a x-momentum balance from $0 \to 3$.

- We know no force is applied $0 \to 1$ and $2 \to 3$: only force present $0 \to 3$ is the disk.
- x-momentum balance $0 \to 3$ is : $M_{x,out} - M_{x,in} = F_x$
- $\dot{m} = const$: therefore $F_x = \dot{m}(U_3 - U_0)$
- Choose $\dot{m} = \frac{\pi D^2}{4}\rho U_1$, then $F_x = \frac{\pi D^2}{4}\rho U_1(U_3 - U_0)$

Comparing from Bernoulli: $F = \frac{\pi D^2}{8}\rho(U_3^2 - U_0^2)$

- $2U_1(U_3 - U_0) = (U_3^2 - U_0^2)$ must be true, which means that..
- $U_1 = \frac{U_3+U_0}{2}$. Half the speed increase occurs upstream of the propeller.

Thrust generated, Power consumed and obtained, the theoretical efficiency.

Force Derivation

- We derived the Force (thrust) is $F = \dot{m}(U_3 - U_0)$, and that $\dot{m} = \frac{\pi D^2}{4}\rho U_1$.
- We found that $U_3 = 2U_1 - U_0$ gives:
- Force: $F = \frac{\pi D^2}{4}\rho U_1(2(U_1 - U_0)) = \frac{\rho \pi D^2}{2} U_1^2 \left(1 - \frac{U_0}{U_1}\right)$.

Efficiency Derivation

- Power consumed : $FU_1 = \frac{\rho \pi D^2}{2} U_1^3 \left(1 - \frac{U_0}{U_1}\right)$.
- Power useful : Thrust × Velocity of advance : FU_0.
- Perfect (lossless) efficiency : Useful/Consumed : $\eta_{perfect} = \frac{FU_0}{FU_1} = \frac{U_0}{U_1}$.

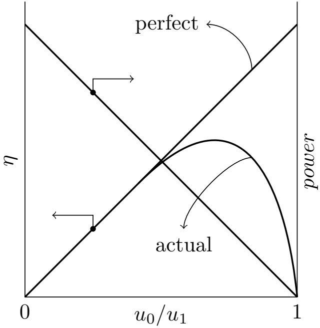
Theoretical characteristics of propellers.

Thrust

- Thrust (force): $F = \frac{\rho \pi D^2}{2} U_1^2 \left(1 - \frac{U_0}{U_1}\right)$
- Proportional to $D^2, U_1^2, \frac{U_0}{U_1}$, which means.
- Large rotors and high wind speeds at the rotor required, as is the streamline curvature.

Power and Efficiency Trade off

- Power consumed $D^2, U_1^3, \frac{U_0}{U_1}$ - highly non-linear with wind speed.
- Theoretical Efficiency directly proportional to $\frac{U_0}{U_1}$, 100% efficiency is impossible.
- The more power you add via reducing $\frac{U_0}{U_1}$, the less efficiently you do so !
- Bigger slower propellers are more efficient.



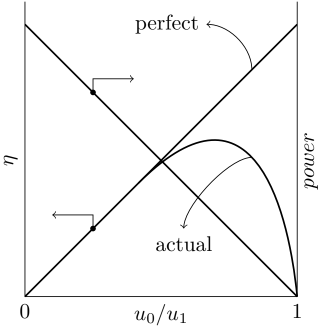
Actual Efficiency of propellers.

In reality there are losses due to:

- Frictional effects on the propeller blade surface.
- Rotational energy imparted to the fluid when it goes through the propeller.
- Pressure variations on the surfaces of our 'black box'.

The power losses are (roughly) proportional to the momentum entering the propeller,

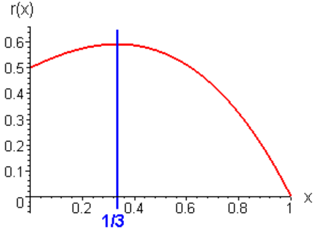
- $P_{LOSS} \propto \dot{m} U_1^2$ and therefore...
- $P_{LOSS} = C \dot{m} U_1^2$.



Therefore the actual efficiency is:

- $\eta_{actual} = \frac{FU_0}{FU_1 + P_{LOSS}}$.
- Typical aircraft have efficiencies of 0.85 - very impressive !

Theoretical characteristics of wind turbines.



Most Propeller Conclusions Apply Here

- $F \propto D^2$.
- $F \propto (U_3^2 - U_0^2)$.

Power and Efficiency is different

- Power available is $FU_0 = \frac{1}{2}\rho A_1 {U_0}^3$
- Power extracted is FU_1
- $\eta_{perfect} = \frac{FU_1}{\frac{1}{2}\rho A_1 U_0^3} = \frac{1}{2}\left(1 - \frac{U_3}{U_0}\right)\left(1 + \frac{U_3}{U_0}\right)^2$
- Known as the Betz Limit (no need to be able to prove this).
- Figure shows efficiency versus U_3/U_0, 59% is the maximum possible.
- Impacted by gusts, frictional, mechanical and electrical losses.

Summary and Suggested Reading

Lecture Summary

- Derived actuator disk theory, using the Bernoulli equation (away from the disk) and the Momentum equation (across it).
- Showed that to extract momentum from or deliver momentum to the fluid, the streamlines must expand or compress across the disk.
- Power output is improved in all cases by using larger blades.
- Losses are due to frictional, rotational and pressure fluctuation effects.
- The best propellers have efficiencies of 85% .
- The theoretical efficiency of a wind turbine, and its maximum efficiency (the Betz Limit) is 0.59.

Suggested Further Reading

- Cengel and Cimbala, ’Fluid Mechanics: Fundamentals and Applications’, 2^{nd} ed, SI units, Page 822 onwards (wind turbines).
- Shrimpton, Section 4.10.

Suggested Further Study

- Material covers chapters 9 and 10.
- Try 10.4

Lecture 27: The Steady Flow Energy Equation.

Last Lecture we covered:

- Derived actuator disk theory, using the Bernoulli equation (away from the disk) and the Momentum equation (across it).
- Showed that to extract momentum from or deliver momentum to the fluid, the streamlines must expand or compress across the disk.
- Power output is improved in all cases by using larger blades.
- Losses are due to frictional, rotational and pressure fluctuation effects.
- The best propellers have efficiencies of 85% .
- The theoretical efficiency of a wind turbine, and its maximum efficiency (the Betz Limit) is 0.59.

This lecture we are going to cover:

- Derive a general energy equation, for steady flows, based on the First Law of Thermodynamics.
- It is really the (Rate of Change of the) First Law for Flow Systems.
- We do this by starting from a (Lagrangian) system description, and apply the Reynolds Transport Theorem to convert it to an (Eulerian) control volume description.
- Then we simplify the SFEE for classic engineering sub-systems.

From the First Law of Thermodynamics (system) to the SFEE (control volume)

The First Law of Thermodynamics (for a system, fixed mass) is an energy conservation balance

$$\begin{bmatrix} \textit{thermal} \\ \textit{energy transfer across} \\ \textit{system boundary } (Js^{-1}) \end{bmatrix} - \begin{bmatrix} \textit{work the} \\ \textit{system does on its} \\ \textit{environment } (Js^{-1}) \end{bmatrix} = \begin{bmatrix} \textit{change} \\ \textit{of energy of the} \\ \textit{system mass } (Js^{-1}) \end{bmatrix}$$

- Sign convention for thermal energy transfer is positive into the system.
- Sign convention for work energy transfer is positive out of the system.
- $Q_{12} - W_{12} = E_{u,12}$.

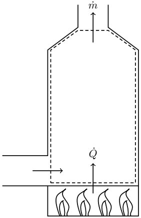
The SFEE is the First Law, only a *Rate of Energy Change* of a *control volume*

- Because the SFEE is a energy rate of change equation, terms like Q_{12} for a system become $\dot{Q}$ for a CV
- A control volume can have 'energy' flowing in/out of it by convection and diffusion.
- In the SFEE conduction (diffusion) of energy through the control volume surface is $\dot{Q}$.
- The convective transfer of energy by the flow is in the term '$\dot{E}_u$'.
- We have to include all forms of energy (cf the Bernoulli equation)
- Work $\dot{W}$ is 'non-fluid' energy transfer (power). For example electrical work increasing the fluid pressure in a pump.

SFEE : Non-flow energy terms (Q and W)

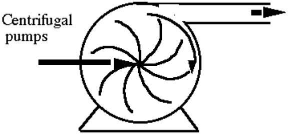
Q - Heat Conduction - non-flow heat transfer

- Classic examples include boilers, heat exchangers.
- Anything where energy is added/removed from the fluid due heat transfer and...
- ....not due to fluid convection into/out of the CV
- Q>0 when *adding* energy.



W - Non-thermal Non-flow energy transfer

- Classic examples include pumps, turbines, electrical heaters.
- Anything where energy is added/removed from the fluid not due heat transfer and...
- ...not due to fluid convection into/out of the CV
- W>0 when energy is *extracted* from the fluid (ie a turbine)



SFEE : Flow energy terms

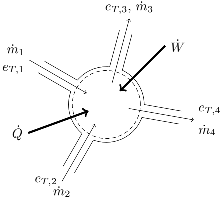
Energy Conservation due to convection and accumulation in a CV

- For the system First Law, $E_{u,12}$ represented the change in internal energy in the system from state 1 to 2
- For the SFEE, we need to consider other forms of energy, and define a rate of change of energy.
- The rate of change of a system variable is defined as the material derivative. So if E_T is our energy in the system, the rate of change is $\frac{D}{Dt}E_T$
- To define this for a control volume, we use the Reynolds Transport Theorem

$$\frac{D}{Dt}E_T = \frac{\partial}{\partial t}\int \rho e_T \delta V + \int_A \rho e_T \overrightarrow{U} \bullet \delta \overrightarrow{A}$$

- Note we are going from Extensive (system) to intensive (control volume) $E_T = \int \rho e_T \delta V \approx \rho e_T \Delta V$.
- We are not dealing with unsteady energy problems in this module, hence

$$\frac{D}{Dt}E_T = \int \rho e_T \overrightarrow{U} \bullet \delta \overrightarrow{A}.$$

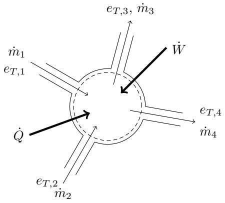


This gives

$$\dot{Q} - \dot{W} = \int \rho e_T \overrightarrow{U} \bullet \delta \overrightarrow{A}$$

Simplifying the Flow energy

Simplifying $\int \rho e_T \overrightarrow{U} \bullet \delta \overrightarrow{A}$



- note $\dot{m} = \int \rho \overrightarrow{U} \bullet \delta \overrightarrow{A}$.
- if fluid is *inviscid*, and *uniform* across a CV inlet/outlet then...
- $\dot{m} = \rho U_n A$ and..
- Rate of energy flow across an inlet/outlet is $\dot{m} e_T$ and..
- SFEE : $\dot{Q} - \dot{W} = \sum_{out} \dot{m} e_T - \sum_{in} \dot{m} e_T$

Decomposing the total energy to components

The last stage is to decompose our total energy in its components:
$e_T = e_u + e_k + e_p + e_g$ where:

- $e_u = C_V T$: specific internal energy.
- $e_k = \frac{U^2}{2}$: kinetic energy per unit mass.
- $e_p = \frac{p}{\rho}$: pressure energy per unit mass.
- $e_g = gz$: potential energy per unit mass.

The conceptual leap required is these energies are convected by the flow velocity.

Dealing with energy flows

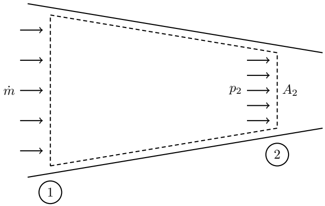
$$\dot{Q} - \dot{W} = \int (C_V T + \frac{U^2}{2} + \frac{p}{\rho} + gz)\rho \overrightarrow{U} \bullet \delta \overrightarrow{A}.$$

- These energies are convected into and out of the control volume by the flow.
- This seems obvious for thermal and kinetic energy, but less obvious for potential and pressure energy.
- For potential energy remember the best way to think about it is the net change in height between an inlet and an outlet of a CV - a waterfall.
- Often internal and pressure energies are added to define the specific enthalpy $e_h = C_V T + \frac{p}{\rho}$. In energy-flow problems - use enthalpy !
- BE CAREFUL with units when mixing kinetic and thermal energy !
- The pressure energy (pressure work or flow work as it is sometimes called) is added to $\dot{W}$ in some texts. Strictly, this is not correct (cf slide 2).
- .....More on this on the next slide.

Explanation of Pressure energy inflow/outflow.

We use the idea that work = energy = force × distance

- Examine the flow through a converging section.
- At 2, the outlet, the fluid inside the CV pushes on the fluid outside the CV with a force $F_2 = p_2 A_2$.
- During a time interval δt the fluid displacement is $\delta x_2 = U_2 \delta t$.
- So the amount of flow work is $\delta e_2 = F_2 \delta x_2 = p_2 A_2 U_2 \delta t$.
- The rate of flow work is therefore $\frac{\delta e_2}{\delta t} = p_2 A_2 U_2 = \frac{p_2}{\rho} \rho A_2 U_2 = \frac{p_2}{\rho} \dot{m}$.



This energy transfer is positive because the work is being doing by the CV fluid on the outside.

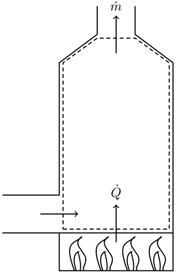
At 1, the inlet, the term is negative because the fluid outside of the CV is doing work on the fluid inside the cv.

Example : Inviscid Flow Through a Boiler.

Simplifying the SFEE

- Inviscid form of the SFEE:
$\dot{Q} - \dot{W} = \sum_{out} \dot{m}(e_h + \frac{U^2}{2} + gz) - \sum_{in} \dot{m}(e_h + \frac{U^2}{2} + gz)$.
- CV is drawn to enclose only the fluid which is being heat by a burner
- 2 ports.
- $\dot{W} = 0$.
- Potential energy negligible.
- Kinetic energy negligible.
- Q is positive (adding energy to the fluid in the boiler).

SFEE simplifies to: $\dot{Q} = \dot{m}(e_{h,out} - e_{h,in})$.

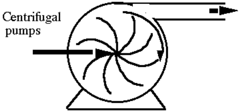


Example : Inviscid Flow Through a Pump/Turbine/Compressor.

Simplifying the SFEE

- Inviscid form of the SFEE: $\dot{Q} - \dot{W} = \sum_{out} \dot{m}(e_h + \frac{U^2}{2} + gz) - \sum_{in} \dot{m}(e_h + \frac{U^2}{2} + gz)$.

- Pumps are generally use to increase the pressure of the fluid.
- 2 ports.
- $\dot{Q} = 0$
- Potential energy negligible.
- Kinetic energy negligible (most of the time).
- Pump/Compressor: W is negative, leading to $-W$ being positive.
- Pump power is defined as flow rate × pressure drop.



SFEE simplifies to: $-\dot{W} = \dot{m}(e_{h,out} - e_{h,in})$

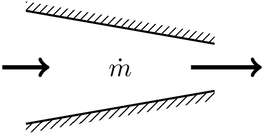
For incompressible fluids density is constant (and not a function of T). Furthermore, when there are relatively small temperature changes in the pump, then:

$$-\dot{W} = \frac{\dot{m}}{\rho}(p_{out} - p_{in})$$

Example : Inviscid Flow Through a Nozzle.

Simplifying the SFEE

- Inviscid form of the SFEE: $\dot{Q} - \dot{W} = \sum_{out} \dot{m}(e_h + \frac{U^2}{2} + gz) - \sum_{in} \dot{m}(e_h + \frac{U^2}{2} + gz)$.
- Nozzles are devices for increasing the flow velocity at the exit - examples being end of a hosepipe to a jet engine exhaust. Often the exit pressure and temperature are at atmospheric conditions.
- 2 ports.



- $\dot{Q} = \dot{W} = 0$
- Potential energy negligible.
- Sometimes you can ignore the inlet KE (as here).

SFEE simplifies to: $\frac{U_{out}^2}{2} = (e_{h,in} - e_{h,out})$

For incompressible fluids density is constant (and not a function of T). Furthermore, when there are relatively small temperature, then, i.e. the Bernoulli equation is recovered.

$$\frac{\rho U_{out}^2}{2} = (p_{in} - p_{out})$$

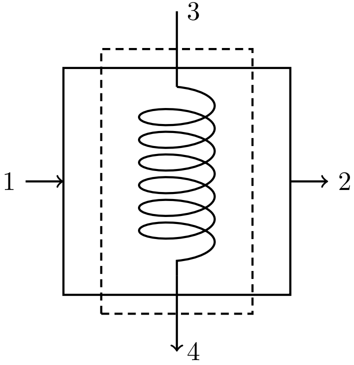
Example : Inviscid Flow Through a Heat Exchanger.

Simplifying the SFEE

- Inviscid form of the SFEE: $\dot{Q} - \dot{W} = \sum_{out} \dot{m}(e_h + \frac{U^2}{2} + gz) - \sum_{in} \dot{m}(e_h + \frac{U^2}{2} + gz)$.
- Heat exchangers transfer heat from a hot stream to a cold stream, without mixing.
- 4 ports.
- $\dot{W} = \dot{Q} = 0$ (no heat transfer across the cv surface).
- Potential, kinetic energy negligible.

SFEE simplifies to:

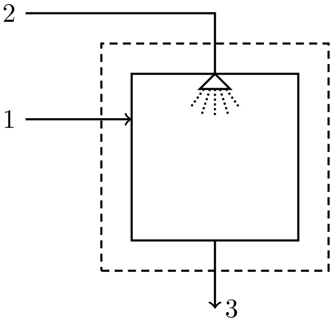
$$\dot{m}_1 e_{h,1} + \dot{m}_3 e_{h,3} = \dot{m}_2 e_{h,2} + \dot{m}_4 e_{h,4}$$



Example : Inviscid Flow Through a Mixing Chamber.

Simplifying the SFEE

- Inviscid form of the SFEE: $\dot{Q} - \dot{W} = \sum_{out} \dot{m}(e_h + \frac{U^2}{2} + gz) - \sum_{in} \dot{m}(e_h + \frac{U^2}{2} + gz)$.
- Mixing chambers transfer heat and mass from a hot stream to a cold stream to another stream, for instance a shower head.
- 3 ports.
- $\dot{W} = \dot{Q} = 0$ (no heat transfer across the cv surface).
- Potential, kinetic energy negligible.



SFEE simplifies to:

$$\dot{m}_1 e_{h,1} + \dot{m}_2 e_{h,2} = \dot{m}_3 e_{h,3}$$

Lecture 27: Summary and Suggested Reading

Lecture Summary

- Derived a control volume based energy equation appropriate for open processes
- Contains no other assumptions other than the condition of steady flow
- Flows through the control surface can be simplified if inviscid fluid assumed
- Provided a set of examples of typical engineering plant and showed the key features of each.

Suggested Further Reading

- Crowe et al. 'Engineering Fluid Mechanics', 9^{th} ed. SI version, Section 7.1-7.5.
- Shrimpton, Section 4.7.

Suggested Further Study

- Workbook Chapter 11 can now be completed
- worked solution 11.4

Lecture 28: Practical Engine Cycles: Gas Turbine Engines (Revisited).

Last Lecture we covered:

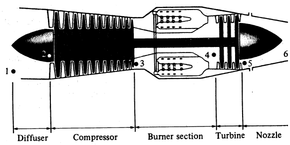
- Derived a control volume based energy equation appropriate for open processes
- Contains no other assumptions other than the condition of steady flow
- Flows through the control surface can be simplified if inviscid fluid assumed
- Provided a set of examples of typical engineering plant and showed the key features of each.

This lecture we are going to cover:

- Describe jet engine operation.
- Use of the Steady Flow Energy Equation and Force-Momentum Equation.
- Open Cycle - control volume description.
- Similar to the Brayton Cycle used for closed cycles (systems).

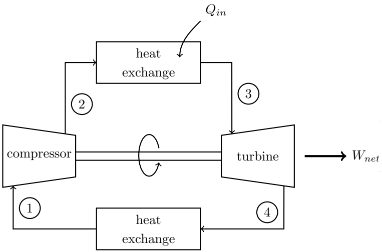
Jet Engine Basic Characteristics.

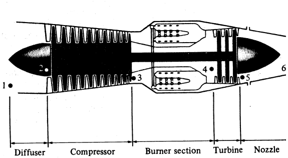
- Diffuser slows gas stream down (kinetic to pressure energy).
- Compressor pressurises the gas.
- Burner section adds (thermal) energy.
- Turbine extracts sufficient power to drive the compressor.
- Nozzle converts the remaining pressure energy to kinetic energy.



- Thrust (force) developed by the aircraft is the difference of momentum in the inlet air, and the momentum in the exhaust. $F = \dot{m}(U_{out} - U_{in})$.
- These velocities are relative to the aircraft speed. In still air U_{in} is the aircraft speed.
- The power is the force × distance the force acts per unit time, i.e. $P = FU_{in} = \dot{m}(U_{out} - U_{in})U_{in}$.
- The efficiency is the useful propulsive power per unit rate of energy input from combustion. $\eta = \frac{P}{\dot{Q}_{in}}$

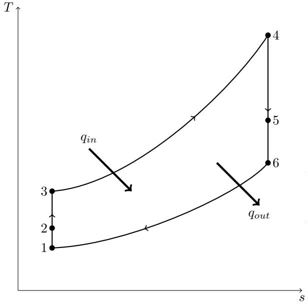
Jet Engine Open Cycle and Brayton Closed Cycle comparison.

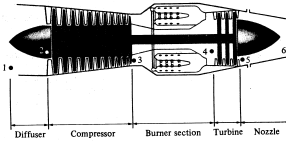




- In the Brayton Cycle the working fluid goes around the cycle - In the Jet Engine the point is to expel fluid from the exhaust.
- In the Brayton Cycle the Turbine drives the compressor, but the aim of the Brayton Cycle is to maximise Turbine output for useful work.
- In the Jet Engine the turbine is only just sufficient to power the compressor and the aim is to conserve turbine exit energy for conversion in the nozzle section. The net work of a jet engine is zero.
- Because some compression occurs in the diffuser section of a jet engine the pressure ratios are generally higher (10-25) than for a Brayton Cycle.

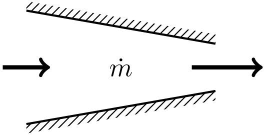
Jet Engine Open Cycle: Temperature- Entropy Diagram.





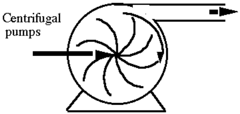
- Process 1-2: Isentropic Compression in a Diffuser.
- Process 2-3: Isentropic Compression in a Compressor.
- Process 3-4: Constant Pressure heat addition due to combustion.
- Process 4-5: Isentropic Expansion in a Turbine.
- Process 5-6: Isentropic Expansion in a Nozzle.
- Process 6-1: (Theoretical) - this is the energy lost due to low temperature gases being ingested by the engine and high temperature gases being expelled.

Jet Engine Cycle Components: Nozzle



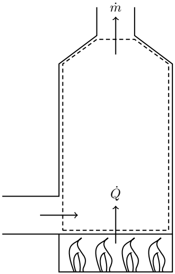
- In a nozzle the area decreases, and the fluid speeds up.
- It is assumed that the process is isentropic, and the air KE at the inlet of the nozzle is negligible.
- Inviscid form of the SFEE: $\dot{Q} - \dot{W} = \sum_{out} \dot{m}(e_h + \frac{U^2}{2} + gz) - \sum_{in} \dot{m}(e_h + \frac{U^2}{2} + gz)$.
- 2 ports, $\dot{Q} = \dot{W} = 0$, Potential energy, inlet KE negligible.
- SFEE simplifies to: $e_{h,in} = e_{h,out} + \frac{U_{out}^2}{2}$.
- The diffuser section is simply the inverse of this process.

Jet Engine Cycle Components: Compressor.



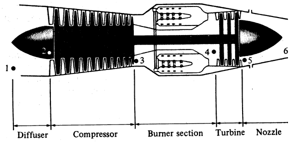
- In a compressor the fluid is compressed, again assumed to be is-entropic.
- Inviscid form of the SFEE: $\dot{Q} - \dot{W} = \sum_{out} \dot{m}(e_h + \frac{U^2}{2} + gz) - \sum_{in} \dot{m}(e_h + \frac{U^2}{2} + gz)$.
- 2 ports, $\dot{Q} = 0$, Potential/Kinetic energy negligible (most of the time).
- Compressor: W is negative, leading to W being positive.
- SFEE simplifies to: $-\dot{W} = \dot{m}(e_{h,out} - e_{h,in})$.
- The turbine stage is simply the inverse of this process, and the power output is equal to compressor input.

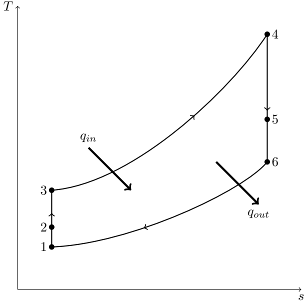
Jet Engine Components: Burner Section.



- Inviscid form of the SFEE: $\dot{Q} - \dot{W} = \sum_{out} \dot{m}(e_h + \frac{U^2}{2} + gz) - \sum_{in} \dot{m}(e_h + \frac{U^2}{2} + gz)$.
- CV is drawn to enclose only the fluid which is being heat by a burner, 2 ports.
- $\dot{W} = 0$, Potential/Kinetic energy negligible. Q is positive (adding energy to the fluid in the boiler).
- SFEE simplifies to: $\dot{Q} = \dot{m}(e_{h,out} - e_{h,in})$.

Jet Engine: Typical Efficiency and (theoretical) losses.





- $P = FU_{in|} = \dot{m}(U_{out} - U_{in})U_{in}$.
- $\eta = \frac{P}{\dot{Q}_i n} \approx 22\%$.
- Excess KE $\approx$ 33% (KE of the exhaust gas relative to the ground).
- Excess enthalpy $\approx$ 45% (Temperature rise of the exhaust gas over ambient air.

Summary and Suggested Reading

Lecture Summary

- Comparison of Open Cycle Jet Engine versus Closed Cycle Brayton.
- Definition of Jet Engine Thrust, Power and Efficiency.
- Definition and quantitative description of the jet engine processes.
- Analysis of the Energy Rate budgets showing sources and levels of inefficiency.

Suggested Further Reading

- Cengel and Boles, Section 8-11
- Shrimpton, Section 4.11.

Suggested Further Study

- Workbook Chapter 11 can now be completed
- worked solution 11.13

Lecture 29: The Steady Mechanical Energy Equation.

Last Lecture we covered:

- Comparison of Open Cycle Jet Engine versus Closed Cycle Brayton.
- Definition of Jet Engine Thrust, Power and Efficiency.
- Definition and quantitative description of the jet engine processes.
- Analysis of the Energy Rate budgets showing sources and levels of inefficiency.

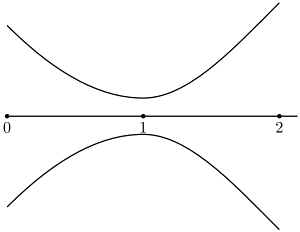
This lecture we are going to cover:

- Derive a sub-set of the Steady Flow Energy Equation, for isothermal fluid.
- Strictly this is the Steady Mechanical Energy Equation. It is however often called the 'Extended Bernoulli Equation'.
- This SMEE equation is used in process equipment design using pipework. It is an isothermal viscous flow energy conservation equation.
- Show that you cannot derive the Extended Bernoulli equation from the Bernoulli equation (but you can derive Bernoulli equation from the extended form!).
- Show various examples of the use of the EBE/SMEE.

Reminder: The Bernoulli Equation (inviscid flow along a streamline).

Assumptions by using a streamline as a basis

- The flow must be steady - otherwise streamlines have no meaning.
- There are no viscous forces present.
- We have also assumed that the fluid is incompressible, but that is rather subtle.



Force balance : Euler Equation

- $\rho U_S \frac{dU_S}{ds} + \frac{dp}{ds} + \rho g \frac{dz}{ds} = 0.$
- This tells us that total force change is zero along a stream tube, but it can change form.

Mechanical Energy Balance : Bernoulli Equation

- Integrate w.r.t. S to get a mech energy balance (Energy = Force $\times$ distance)
- $\frac{\rho U_S^2}{2} + p + \rho g z$ = Const.

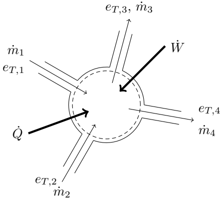
Reminder: Steady Flow Energy Equation.

Energy Conservation over a control volume, assuming steady flow

$\dot{Q} - \dot{W} = \int_A \rho e_T \overrightarrow{U} \bullet \delta \overrightarrow{A}$

If inviscid then $\dot{Q} - \dot{W} = \sum_{out} \dot{m} e_T - \sum_{in} \dot{m} e_T$

- $\dot{Q}$: Heat transfer rate (positive into CV volume, across surface).
- $\dot{W}$: Work transfer rate (positive out of CV, across surface).



$e_T = e_u + e_k + e_p + e_g$ where:

- $e_u = C_V T$: Specific internal energy.
- $e_k = \frac{U^2}{2}$: Kinetic energy per unit mass.
- $e_p = \frac{p}{\rho}$: Pressure energy per unit mass.
- $e_g = gz$: Potential energy per unit mass

Deriving the Steady Mechanical Energy Equation.

Typically, we are interested in pipe networks

- assume two ports
- 1-D flow (in the pipe direction)

$$\dot{Q} - \dot{W} = \int_{out} (C_V T + \frac{U^2}{2} + \frac{p}{\rho} + gz) \rho U \delta A - \int_{in} (C_V T + \frac{U^2}{2} + \frac{p}{\rho} + gz) \rho U \delta A$$

We know that $\dot{m} = \int \rho U \delta A$, and that $\dot{m}_{out} = \dot{m}_{in}$ so most of these terms simplify:

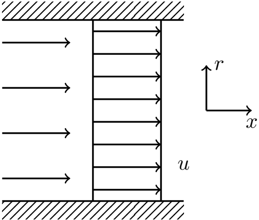
$$\dot{Q} - \dot{W} = \dot{m}(C_V T + \frac{p}{\rho} + gz)_{out} + \frac{1}{2} \int_{A_{out}} U^2 \rho U \delta A - \dot{m}(C_V T + \frac{p}{\rho} + gz)_{in} + \frac{1}{2} \int_{A_{in}} U^2 \rho U \delta A$$

KE Term

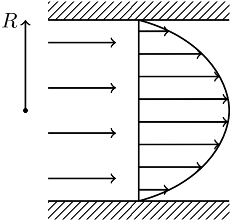
- If a uniform flow (i.e. U did not depend on the pipe radial direction) then the KE term is $\frac{\dot{m} U^2}{2}$.
- This is only true if the fluid is inviscid, which is never true in practice, and viscous forces are ALWAYS important near a wall.
- This is the main reason you CANNOT use the Bernoulli Equation to work out the pressure drop in a pipe (or anything else with a wall).

Kinetic Energy Correction Factor.

(a) a uniform (inviscid) and (b) shows a real (viscous) velocity profile, same mass flow



$KE\ (a) = \frac{\dot{m}U^2}{2}$



$KE\ (b) = \int_A \frac{U^2}{2}\rho U \delta A$

The KE correction factor fix

- factor is defined $\alpha = \frac{KE(b)}{KE(a)}$
- $KE(b) = \alpha KE(a) = \alpha \dot{m}\frac{\overline{U}^2}{2}$.
- $\alpha = 1$ (uniform flow), $\alpha = 2$ (laminar flow), $2 > \alpha > 1$ (turbulent flow).
- Note: It must be applied to each flow boundary individually.

Further Simplifications to obtain the SMEE.

Incorporating the KE correction term

$$\frac{\dot{Q}}{\dot{m}} - \frac{\dot{W}}{\dot{m}} = (C_V T + \frac{p}{\rho} + gz + \frac{1}{2}\alpha\overline{U}^2)_{out} - (C_V T + \frac{p}{\rho} + gz + \frac{1}{2}\alpha\overline{U}^2)_{in}$$

Other than steady flow, 2 ports, no other assumptions made.

The SMEE is an isothermal viscous flow equation, so we re-arrange the equation to group the thermal components.

$$-\frac{\dot{W}}{\dot{m}} = (\frac{p}{\rho} + gz + \frac{1}{2}\alpha\overline{U^2})_{out} - (\frac{p}{\rho} + gz + \frac{1}{2}\alpha\overline{U^2})_{in} + \left[C_V(T_{out} - T_{in}) - \frac{\dot{Q}}{\dot{m}}\right]$$

The thermal terms are an irreversible temperature rise due to viscous dissipation, and if Q is non-zero, the transfer of that thermal energy through the pipe wall.

This energy loss is real and significant and must be considered in design calculations for viscous flows near walls and through engineering plant.

The thermal terms as presented here are not practically useful, empirical models are used instead.

Models for viscous losses in the SMEE.

Viscous losses : wall friction and fittings

$$\left[C_V(T_{out}-T_{in})-\frac{\dot{Q}}{\dot{m}}\right] \implies f\frac{L}{d}(\tfrac{1}{2}\overline{U}^2)+\sum_n k_n(\tfrac{1}{2}\overline{U}^2)$$

- Mostly the empirical model coefficients are obtained by lab experiments ensuring dynamic similarity.
- Wall friction requires an empirical coefficient (f, next slide) for a pipe of length L and diameter d.
- There are also many types of fitting, bends, joints etc that cause losses.
- The losses ALL arise due to viscous dissipation of kinetic energy.

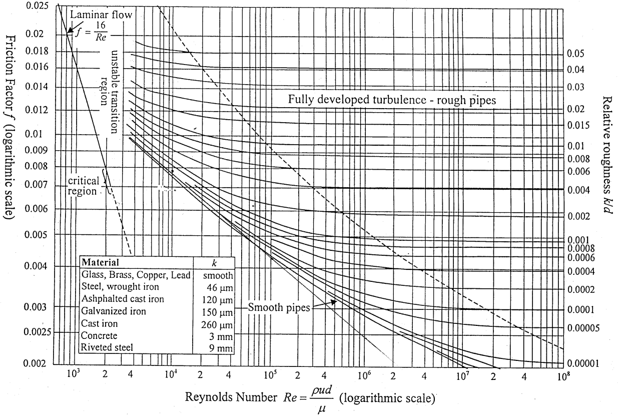
This gives the final form of the SMEE or the 'Extended Bernoulli Equation', EBE.

$$-\frac{\dot{W}}{m}=(\frac{p}{\rho}+gz+\tfrac{1}{2}\alpha\overline{U}^2)_{out}-(\frac{p}{\rho}+gz+\tfrac{1}{2}\alpha\overline{U}^2)_{in}+f\frac{L}{d}(\tfrac{1}{2}\overline{U}^2)+\sum_n k_n(\tfrac{1}{2}\overline{U}^2)$$

Note: Although it is often called the 'Extended Bernoulli Equation' it is not derived or extended from the Bernoulli Equation.

Losses due to friction at the pipe wall (moody diagram).

Moody Diagram defines the friction coeft versus Pipe Re



Material	k
Glass, Brass, Copper, Lead	smooth
Steel, wrought iron	46 µm
Ashphalted cast iron	120 µm
Galvanized iron	150 µm
Cast iron	260 µm
Concrete	3 mm
Riveted steel	9 mm

- f is obtained by working out the Pipe Reynolds number and deciding laminar/turbulent.
- A full explanation of pipe roughness and turbulence is given in Section 4 - assume smooth for now!

MORE simplifications to recover the Bernoulli Equation.

Starting from our SMEE/EBE

$$-\frac{\dot{W}}{\dot{m}} = (\frac{p}{\rho} + gz + \frac{1}{2}\alpha\overline{U}^2)_{out} - (\frac{p}{\rho} + gz + \frac{1}{2}\alpha\overline{U}^2)_{in} + f\frac{L}{d}(\frac{1}{2}\overline{U}^2) + \sum_n k_n(\frac{1}{2}\overline{U}^2)$$

- No work transfers
- No viscous losses
- Uniform flow

$$(\frac{p}{\rho} + gz + \frac{1}{2}U^2)_{out} = (\frac{p}{\rho} + gz + \frac{1}{2}U^2)_{in}$$

- You can get from SFEE → SMEE/EBE → Bernoulli Equation.
- You CANNOT start from the BE and derive the EBE. Be careful: Several textbooks do exactly this !

Summary and Suggested Reading

Lecture Summary

- Started from the general steady flow energy equation (SFEE).
- Simplified it (2 ports, uni-directional flow in a pipe).
- Defined a Kinetic Energy Correction Factor.
- Assumed isothermal conditions, thermal terms account for viscous energy losses.
- Introduced models for fittings and pipe friction.
- Showed you can simplify to the Bernoulli Equation, but not start from there to obtain the Extended Bernoulli Equation (SMEE).

Suggested Further Reading

- Crowe et al. 'Engineering Fluid Mechanics', 9^{th} ed. SI version, Chapter 5.
- Shrimpton, Section 5.5.

Suggested Further Study

- solution to 12.3 provided.

Lecture 30 : Couette Flow - The Simplest Viscous Flow

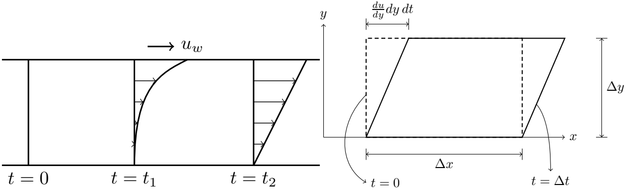
Last Lecture we covered :

- Started from the general steady flow energy equation (SFEE).
- Simplified it (2 ports, uni-directional flow in a pipe).
- Defined a Kinetic Energy Correction Factor.
- Assumed isothermal conditions, thermal terms account for viscous energy losses.
- Introduced models for fittings and pipe friction.

This lecture we are going to cover :

- Stress-Strain relationship for a Newtonian *fluid*.
- Viscous effects on fluid motion (shear stress), using couette flow as an example.
- Variations on the classic Couette Flow system to aid our understanding.

Reminder of Newtonian Stress-Strain relationship



$\frac{\text{Deformation in x}}{\text{length in y}}$ /time $\frac{\frac{dU}{dy}\delta y \delta t}{\delta y} / \delta t = \frac{dU}{dy}$

Dimension Check $\frac{dU}{dy} \equiv [LT^{-1}L^{-1}] = [T^{-1}]$

Note :

- The force at the top wall is the shear force per unit area : the shear stress in the x direction
- It is proportional to the velocity gradient in the y direction through the viscosity coefficient thus..
- $\tau = \mu \frac{dU}{dy}$: Newtons Law of Viscosity.

Rate of Momentum Diffusion: Stoke’s first problem.

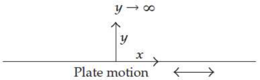
The rate at which momentum diffuses into the fluid from an impulsively started wall to velocity U.

The partial differential equation solved is:

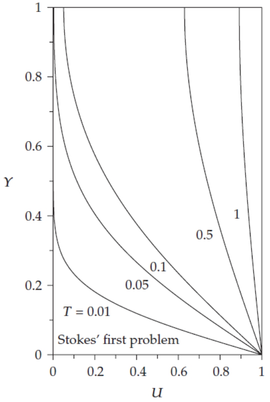
$$\frac{\partial u}{\partial t} = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} = \nu \frac{\partial^2 u}{\partial y^2}$$

And the solution to this is:

$$\frac{u(y,t)}{U} = 1 - erf\left(\frac{y}{2\sqrt{\nu t}}\right)$$



$u = 0$ at $t = 0$
$u = u_0$ for $t > 0$ or
$u = u_0 \cos(\sigma t + \theta)$ for $t > 0$



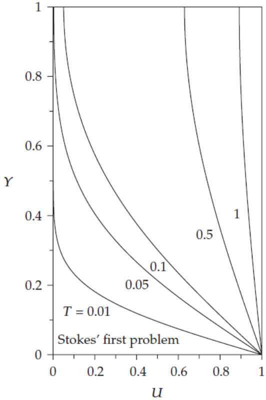
What the solution means...

The solution to this is:
$\frac{u(y,t)}{U} = 1 - erf\left(\frac{y}{2\sqrt{\nu t}}\right).$

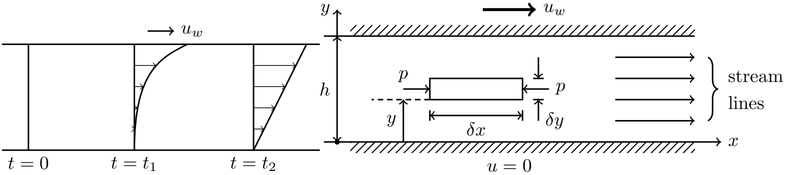
- When $z \approx 1.5$, $erf(z) \approx 0.95$, $\frac{u(y,t)}{U} \approx 0.05$.
- When $\frac{3}{2} = \frac{y}{2\sqrt{\nu t}}$ or $y = 3\sqrt{\nu t}$.
- Or $y \propto \sqrt{\nu}$ and $y \propto \sqrt{t}$.

This shows how the viscosity controls the rate at which momentum in the x-direction is diffused into the fluid in the y-direction.

Larger viscosity fluids diffuse momentum faster.



Analysis of Couette Flow using a Control Volume Analysis.



Key Assumptions

- Two *very large* plates, normal to the y-axis, top one is moving at U_W, bottom one stationary. It is steady
- nothing is changing in the x and z directions so...
- No fluid acceleration in the x direction, *(fully developed and no pressure gradient)*
- No fluid motion in the y direction, so no pressure changes in the y-direction (no hydrostatics here)
- The only force in the fluid is that of viscous shear stress (cf previous slide).

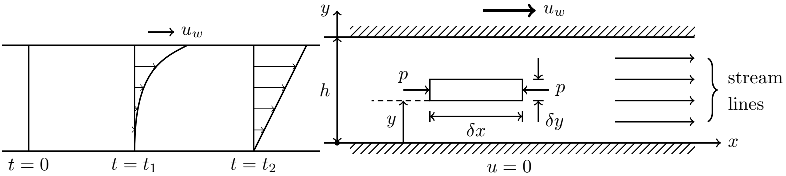
Take a CV, (δx by δy by unit width) say we know what the stress per unit width is at y, so

$$\tau|_y \delta x = \mu \frac{dU}{dy}|_y \delta x$$

We then use a Taylor series to define the stress at $y + \delta y$

$$\tau|_{y+\delta y} \delta x = \left(\tau|_y + \delta y \frac{d}{dy} \tau|_y\right) \delta x = \mu \left(\frac{dU}{dy}|_y + \delta y \frac{d}{dy} \frac{dU}{dy}|_y\right) \delta x$$

Analysis of Couette Flow using a Control Volume Analysis.



- Since there are no other forces on the CV: $\tau|_y = \tau|_{y+\delta y}$ or
- $\mu \frac{dU}{dy} = \mu \left(\frac{dU}{dy}|_y + \delta y \frac{d}{dy} \frac{dU}{dy}|_y \right)$
- $\frac{d^2U}{dy^2} = 0$
- Integrate this twice to get: $U(y) = Ay + B$
- Apply BC's ($U = 0$ at $y = 0$ gives $B = 0$, etc.): $U(y) = U_W \frac{y}{h}$

- Stress is $\tau = \mu \frac{dU}{dy} = \frac{\mu U_W}{h}$, Force is $F = \frac{A\mu U_W}{h}$ and Power is $P = \frac{A\mu U_W^2}{h}$
- Linear variation due to constant shear stress.

Summary and Suggested Reading

Lecture Summary

- Reminded ourselves of the characteristics of the Newtonian fluid.
- For Couette Flow, no other external forces act other than the stress imposed by the moving wall and the no-slip condition.
- Therefore the stress distribution across the plates is constant, and due to the Newtonian stress-strain relationship a linear velocity profile must exist in the fluid.
- Further explored the rate at which momentum diffuses into the fluid from an impulsively started wall and how this is controlled by viscosity.
- Also examined how the frequency and amplitude of an oscillating wall diffuses momentum into the fluid.

Suggested Further Reading

- Crowe et al. '*Engineering Fluid Mechanics*', 9^{th} ed. SI version, Section 9.1.
- Shrimpton, Section 5.2 - 5.3.
- Note: Description of Stoke's first and second problems can be found in I.G. Currie, 'Fundamental Mechanics of Fluids'.

Suggested Further Study

- Start some chapter 12 questions. Solution to 12.2 provided.

Lecture 31 : Poiseulle (Pipe) Flow - Pressure Gradient and Wall Shear

Last Lecture we covered:

- Newtonian Stress-Strain Relationship.
- Stress Balance in Couette flow leads to a linear velocity profile.
- Explored how the viscosity controls the rate at which momentum diffuses into a fluid.

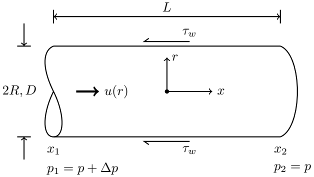
This lecture we are going to cover:

- Pressure driven flow, balanced by wall shear - in a pipe.
- Velocity profile for Laminar flow in a pipe.
- Define the ’friction factor’ (dimensionless wall shear stress) and introduce the Moody diagram.

Pipe Flow, the Basics.

Problem set up and Assumptions

- Assume fully developed in the x-direction (long pipe) - nothing depends on x (inc. dp/dx).
- $\frac{dp}{dx} = \frac{p_1 - p_2}{x_2 - x_1} = -\frac{\Delta p}{L}$: Must be negative as p must fall with x.
- Average (bulk mean) velocity = $\bar{U}$ and volume flow rate $Q = \bar{U}A = \bar{U}\pi R^2$.



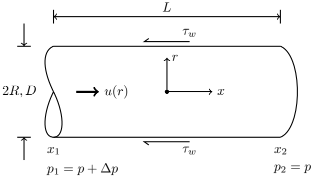
Force Balance using a CV analysis: CV is the pipe inside surface, of length L

- $(p + \Delta p)\pi R^2 - p\pi R^2 - \tau_W 2\pi RL = 0$. Simplifies to : $\frac{\Delta p}{L} = \frac{2\tau_W}{R}$ or
- $\tau_W = -\frac{R}{2}\frac{dp}{dx}$.

This result does not depend on the state of the flow, providing it is fully developed.

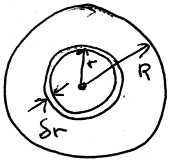
Pipe Flow: Fully developed Laminar flow: Changing y to r coordinates..

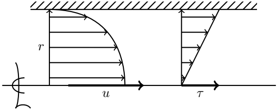
- Exactly the same analysis as before, only now we take a shell of fluid at some point $R > r > 0$ and small length δx.
- Prescribe the pressure change in the x-direction as a Taylor Series (note that $dp/dx < 0$)
$$\left(p - \left(p + \frac{dp}{dx}\delta x\right)\right)\pi r^2 - \tau 2\pi r \delta x = 0.$$



- Now assume that the flow is laminar, hence stress-strain relation: $\tau = \mu \frac{dU}{dy}$.
- y is the distance from the wall, here $y = R - r$ and $\frac{dy}{dr} = -1$,
$\tau = \mu \frac{dU}{dy} = \mu \frac{dU}{dr}\frac{dr}{dy} = -\mu \frac{dU}{dr}$.
- Substitute shear stress definition into force balance gives $\frac{dU}{dr} = \frac{r}{2\mu}\frac{dp}{dx}$

Pipe Flow: Fully developed Laminar flow: Velocity and stress distribution.

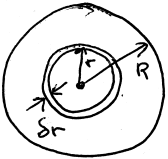


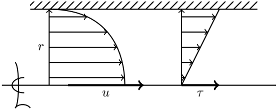


- Integrate $\frac{dU}{dr} = \frac{r}{2\mu}\frac{dp}{dx}$ w.r.t. r gives $U = \frac{r^2}{4\mu}\frac{dp}{dx} + C$.
- Apply B.C. $U = 0$ at $r = R$: $U = -\frac{1}{4\mu}\frac{dp}{dx}(R^2 - r^2) = -\frac{R^2}{4\mu}\frac{dp}{dx}\left(1 - \left(\frac{r}{R}\right)^2\right)$
- This shows that the velocity profile in a laminar fully developed round pipe is parabolic.
- This also reveals the shear stress distribution. $\tau = -\mu\frac{dU}{dr} = -\frac{1}{2}\frac{dp}{dx}r$.
- The stress distribution linearly increases from zero on the centreline to $\tau_W = -\frac{R}{2}\frac{dp}{dx}$ at the wall.

Pipe Flow: Fully developed Laminar flow: Flow rate, max and average (bulk) velocity.

- Velocity Profile is $U = -\frac{R^2}{4\mu}\frac{dp}{dx}\left(1 - \left(\frac{r}{R}\right)^2\right)$.
- Max U occurs at $r = 0$ is $U_{max} = -\frac{R^2}{4\mu}\frac{dp}{dx}$.
- A more common form of the velocity profile is $\frac{U}{U_{max}} = \left(1 - \left(\frac{r}{R}\right)^2\right)$.
- Volume flow rate from integration of $\delta Q = U 2\pi r \delta r$.
- $Q = 2\pi\int_0^R U r dr = -\frac{\pi R^4}{8\mu}\frac{dp}{dx}$.
- Recall $\bar{U} = \frac{Q}{A} = \frac{Q}{\pi R^2} = -\frac{R^2}{8\mu}\frac{dp}{dx} = \frac{U_{max}}{2}$.





Pipe Flow: Fully developed Laminar: Coefficient of friction.

From an engineers point of view, we need to know the pressure drop per unit length of pipe, in order to size a pump.

We can pull all the previous information together to develop a dimensionless 'friction coefficient'.

Using dimensional analysis we might expect that $\tau_W = g(\rho, \mu, \bar{U}, D)$.

This leads to $\frac{\tau_W}{\frac{1}{2}\rho\bar{U}^2} = f = g\left(\frac{\rho\bar{U}D}{\mu}\right)$.

Note this is a functional relationship - f depends on Re is some way. It is also completely general in the sense that nothing has been assumed about the velocity field - this result, what there is of it, applies to laminar and to turbulent flow.

Laminar Flow assumption: Using general definition for wall shear $\tau_W = -\frac{R}{2}\frac{dp}{dx}$ and the specific laminar flow mean velocity $\bar{U} = -\frac{R^2}{8\mu}\frac{dp}{dx}$ we can write a specific relation: $4f = \frac{\tau_W}{\frac{1}{2}\rho\bar{U}^2} = \frac{16}{Re}$.

Prandtl's turbulent friction factor correlation for smooth walls/effect of turbulence.

Recall Laminar Relation: $f = \frac{\tau_W}{\frac{1}{2}\rho\bar{U}^2} = \frac{16}{Re}$

Recall also: $\frac{\Delta p}{L} = \frac{2\tau_W}{R}$ or $\tau_W = \frac{\Delta p}{L}\frac{R}{2}$

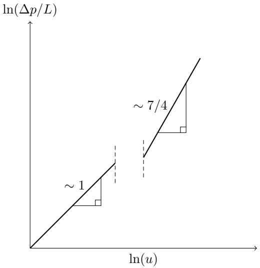
So we could write for laminar and turbulent correlations:

Laminar: $\frac{\Delta p}{L} = \frac{2}{R}\frac{1}{2}\rho\bar{U}^2\frac{16}{Re} = \frac{8\mu}{R^2}\bar{U}$

Prandtl proposed for a *smooth* wall :
$f = \frac{\tau_W}{\frac{1}{2}\rho\bar{U}^2} = 0.079Re^{-1/4}$

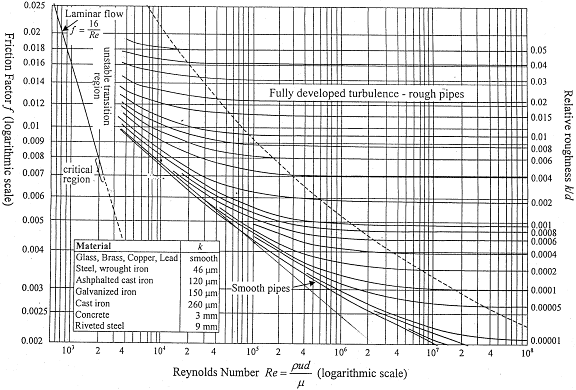
Turbulent: $\frac{\Delta p}{L} = \frac{2}{R}\frac{1}{2}\rho\bar{U}^2(0.079Re^{-1/4}) = g(\bar{U}^{7/4})$

Taking logs of the form: $ln\left(\frac{\Delta p}{L}\right) = ln(\bar{U}) + const$



The Moody Diagram.

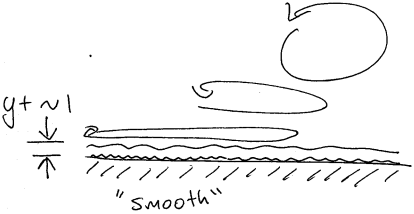
- Laminar friction factor does not depend on the roughness. Laminar friction factor fails at the critical Re for pipe flow.
- Beyond the critical *Re* the pipe flow is turbulent and the wall shear/pressure drop depends on roughness of the pipe as well as the *Re* value.
- For very rough walls, at higher *Re*, the turbulence depends only on the roughness and not on the *Re* at all!

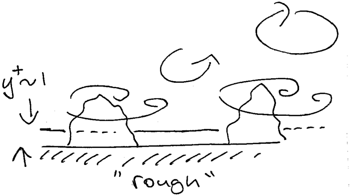


Material	k
Glass, Brass, Copper, Lead	smooth
Steel, wrought iron	46 μm
Ashphalted cast iron	120 μm
Galvanized iron	150 μm
Cast iron	260 μm
Concrete	3 mm
Riveted steel	9 mm

Qualitative explanation of the effect of roughness on pipe pressure drop.

- All real walls have finite roughness. If we assume a Reynolds number based on distance from the wall then.
- $Re_y = \frac{yU}{\nu} \approx y^+$
- Near the wall, viscous forces are important when $Re \approx 1$, and so $y = \nu / U$ and the fluid flow is laminar.
- Further away from the wall, inertial forces are important and the flow is turbulent.
- Note Re_y is a function of U - as U gets bigger, y (the laminar sub-layer gets thinner)
- When the roughness $< y$ then the flow does not 'see' the roughness, has no effect on pressure drop.
- When the roughness $>> y$ it creates eddies by vortex shedding, pressure drop does not depend on Re





Summary and Suggested Reading.

Lecture Summary

- General Relationship between wall shear stress and pressure drop along the pipe - general relation.
- Derivation of the properties of a laminar flow in a round pipe.
- General Relationship for the coefficient of friction, Moody Diagram.
- Features and general structure of a turbulent boundary layer.
- Prantl's empirical coefficient of friction for turbulent boundary layers on smooth walls.
- Comparison of pressure drop vs. bulk velocity for laminar and turbulent boundary layers.
- Effect of Roughness on pressure drop for turbulent boundary layers.

Suggested Further Reading

- Crowe et al. '*Engineering Fluid Mechanics*', 9^{th} ed. SI version, Section 10.5 - 10.7.
- Shrimpton, Section 5.4.

Suggested Further Study

- Start some chapter 12 questions. solution to 12.2 provided.

Lecture 32: External Boundary Layers

Last Lecture we covered:

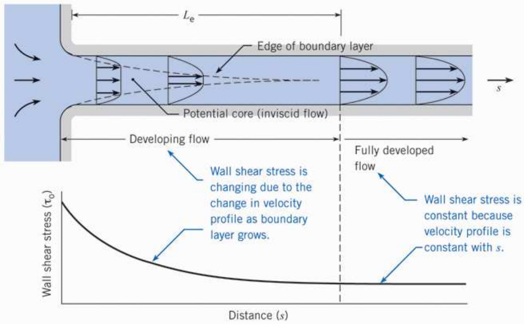
- Laminar and Turbulent Pipe Flow.
- General relationship between wall shear and pressure drop, coefficient of friction.
- Specific relationships for pressure drop laminar and turbulent flow
- Effect of roughness on turbulent flows.

This lecture we are going to cover:

- Boundary Layers, general properties
- The displacement (mass defect) thickness
- The momentum thickness
- Example : Using the momentum thickness to work out the drag on objects moving through fluids.

External Boundary Layers: Link to Internal Flows.

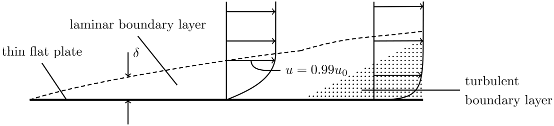
- At the entrance to a pipe, a boundary layer will develop and diffuse the effect of the wall boundary into the flow.
- Boundary layers from the walls will meet and then the flow will be fully developed.



- The shear stress is initially high near the entrance due to the thinness of the boundary layer.
- At fully developed conditions (by definition) the wall shear is not a function of position.

External Boundary Layers: General Characteristics.

- The boundary layer is a thin layer close to a solid surface where the velocity of the fluid falls to zero.
- Outside the boundary layer the fluid can often be treated as inviscid.
- Viscous *friction* drag due to shear stresses occurs solely in the boundary layer.

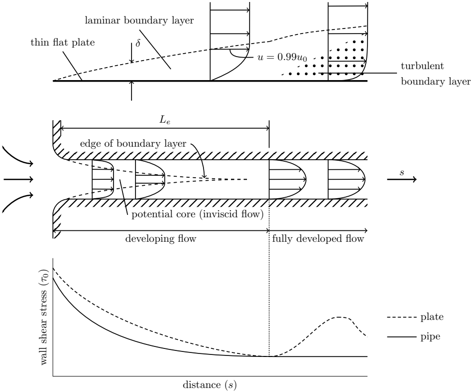


- At the wall a no-slip boundary condition applies.
- The edge of the boundary layer is usually taken to be 99% of the free stream (relative) velocity.
- Boundary layers may be laminar or turbulent.
- Boundary layers may separate, due to an adverse pressure gradient. Generally to be avoided if possible.
- NOT fully developed - everything is changing along the length of the surface *all the time*.

External Boundary Layers: Laminar Versus Turbulent conditions.

$$\tau_W = \mu \frac{\partial U}{\partial y}\bigg|_{y=0}$$

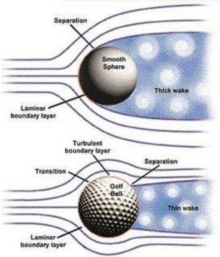
- Laminar initially, eventually a transition to turbulence. Near leading edge du/dy large since dy small.
- As laminar BL thickens, stress reduces.
- After transition, stress level changes because the turbulent near wall velocity profile changes.
- Then again, the *turbulent* boundary layer thickness increases and the wall stress reduces.



Boundary Layers: Properties of Laminar and Turbulent forms.

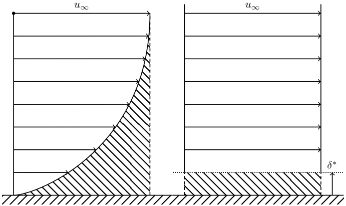
	Laminar	Turbulent
Friction Drag	Low (good)	High (bad)
Separation Resistance	Low (bad)	High (good)

- Laminar and turbulent boundary layers have different properties.
- A golf ball will always produce a turbulent boundary layer, smaller wake.
- Aerofoils need to ensure the flow on the upper surface does not separate during take off and landing but needs to reduce drag at cruise.
- The issue of separation and bluff bodies will be dealt with later, along with *form (pressure)* drag.



Mass and Momentum Budget of a Boundary layer.

Boundary layers are difficult to deal with - we need engineering solutions



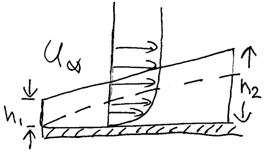
- Displacement thickness (δ^*) deals with the mass defect.
- Momentum thickness (θ) deals with the momentum defect.

- Mass defects : $\rho \int_0^\infty (U_\infty - U)dy = \rho U_\infty \delta^*$ or $\delta^* = \int_0^\infty (1 - \frac{U}{U_\infty})dy$
- Momentum defects : $\rho \int_0^\infty U(U_\infty - U)dy = \rho U_\infty^2 \theta$ or $\theta = \int_0^\infty \frac{U}{U_\infty}(1 - \frac{U}{U_\infty})dy$
- Their ratio is known as the boundary layer 'shape factor' $H = \frac{\delta^*}{\theta}$
- $H = 2.59$ for a laminar boundary layer and 1.3 - 1.6 for a turbulent boundary layer.

This is useful because the drag force on a plate can be defined directly, e.g. $D = \rho U_\infty^2 \theta$.

Example : Showing the Drag to Momentum thickness relation

Find the drag on a flat plate in a stream of fluid of velocity U_∞



- Do a force balance on the CV, top surface a *streamline*
- mass conservation : $\rho U_\infty h_1 = \rho \int_0^{h2} U dy$
- momentum conservation $\int_0^{h_2} \rho U^2 dy - \rho U_\infty^2 h_1 = -D$

The flux of x-momentum out less the flux in must equal the force applied to the control volume, in this case the negative of the drag force. $D = \rho U^2 \theta$

Example Solution.

Mass Conservation : $\rho U_\infty h_1 = \rho \int_0^{h2} U dy$
Momentum Conservation $\int_0^{h_2} \rho U^2 dy - \rho U_\infty^2 h_1 = -D$

- Step 1 : Substitute mass conservation equation into momentum conservation:
- $\rho \int_0^{h_2} U^2 dy - \rho U_\infty \int_0^{h_2} U dy = -D$
- Step 2 : Divide by ρU_∞^2 to give :
- $\frac{D}{\rho U_\infty^2} = \int_0^{h_2} \frac{U}{U_\infty} - \left(\frac{U}{U_\infty}\right)^2 dy = \int_0^{h_2} \frac{U}{U_\infty}\left(1 - \frac{U}{U_\infty}\right) dy = \theta.$
- Therefore: $D = \rho U_\infty^2 \theta$.

And..

- For a length along the plate δx, $\delta D = \tau_W \delta x$
- so in the limit, $\frac{dD}{dx} = \tau_W = \rho U_\infty^2 \frac{d\theta}{dx}$.
- In other words $\frac{d\theta}{dx} = \frac{\tau_W}{\rho U_\infty^2} = 2f$.

Summary and Suggested Reading.

Lecture Summary

- Introduced the features of laminar and turbulent external boundary layers.
- Introduced the concept of mass and momentum thickness of a boundary layer.
- Use of these to work out the friction drag.
- Briefly touched on separation, pressure drag.

Suggested Further Reading

- Crowe et al. '*Engineering Fluid Mechanics*', 9^{th} ed. SI version, Section 9.1 - 9.5.
- Shrimpton, Section 5.6 - 5.7.

Suggested Further Study

- Start some chapter 12 questions.

Lecture 33: Boundary Layer Separation, form (pressure) drag.

Last Lecture we covered:

- Introduced the features of laminar and turbulent boundary layers.
- Introduced the concept of mass and momentum thickness of a boundary layer.
- Use of these to work out the friction drag.
- Briefly touched on separation, pressure drag.

This lecture we are going to cover:

- Adverse pressure gradients and boundary layer separation.
- Form (pressure) and total (form and friction) drag.
- Streamlined and bluff body flows.

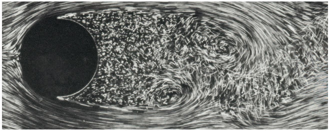
Adverse pressure gradients.

Have examined the wall shear stress (friction drag) for:

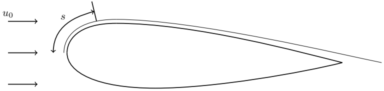
- Couette flow.
- Laminar and turbulent pipe flows.
- Developing boundary layers on flat plates.

But what about the rest of the drag

- Very Low Re (Stokes) flow.
- Low Re flow.
- High Re flow with a turbulent wake but a laminar boundary layer.
- Higher Re flow with a turbulent wake and a turbulent boundary layer.
- Use of roughness to ’trip’ a laminar boundary layer.



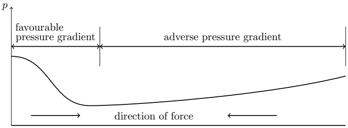
Pressure gradient change along a streamline going round an object.



First consider the streamline starting upstream to a point s along the streamline:

Assuming (???) apply the Bernoulli equation:

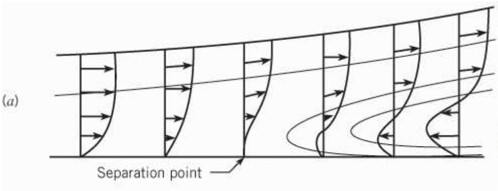
$$P_\infty + \frac{1}{2}\rho U_\infty^2 = P_S + \frac{1}{2}\rho U_S^2$$

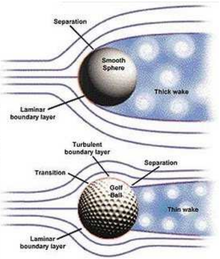


Observations

- The pressure decreases as the streamline as the fluid is accelerated around the nose section of the aerofoil $(dp/ds < 0)$ - boundary moves into the fluid flow direction.
- The pressure increases as the fluid is decelerated after the point of maximum thickness $(dp/ds > 0)$ - boundary moves away from the fluid flow direction.

The Separation Condition.





Boundary layer separation occurs when the local velocity gradient at the wall tends to zero.

Since $\tau_W = \mu \left.\frac{\partial U}{\partial y}\right|_{y=0}$, so does the shear stress.

- Turbulent boundary layers better at delaying the separation point.
- more friction drag, but smaller wake.
- This is why golf balls have dimples.

Pressure as a drag force.

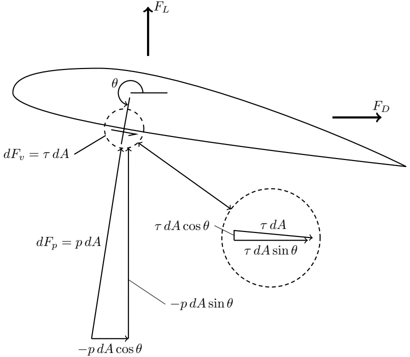
Taking the example of a airfoil

- Both pressure and shear forces exist.
- They both contribute to drag and lift.
- Viscous drag is known as friction drag
- Pressure drag is known as form drag.

An elemental drag force is:

- Viscous Force Magnitude = $\tau\delta A$
- Pressure Force Magnitude =$p\delta A$
- $Drag = \delta F_x = (-pcos\theta + \tau sin\theta)\delta A$
- $Lift = \delta F_y = (-psin\theta - \tau cos\theta)\delta A$

note : $\tau dAcos\theta$ should be negative.

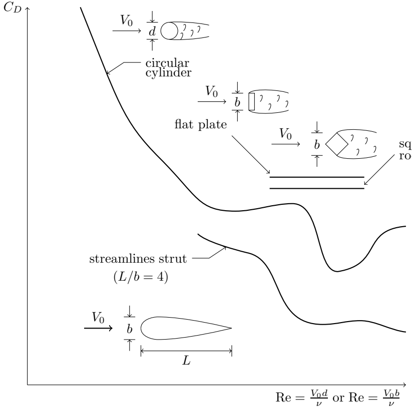


Laminar/Turbulent boundary layers, Streamlining and drag.

$$C_D = \frac{D}{A(\frac{1}{2}\rho U^2)}$$

Cylinder Drag Characteristic

- For the cylinder the linear section is purely friction drag.
- As *Re* increases a wake forms and form drag component rises.
- Boundary layer goes turbulent on $Re \approx 105$.



Other shapes

- Bluff bodies have CD independent of Re as separation always occurs due to surface discontinuity.
- Streamlined objects have lower CD values, they delay separation and reduce the size of the wake once separation does occur.

Reducing Drag in practice.

- For cars, reducing drag below 0.3 is extremely difficult due to other considerations
- For aircraft wings drag reduction must be balanced against lift - drag reduction depends on application.
- For ships friction drag dominates - roughness is the key issue.

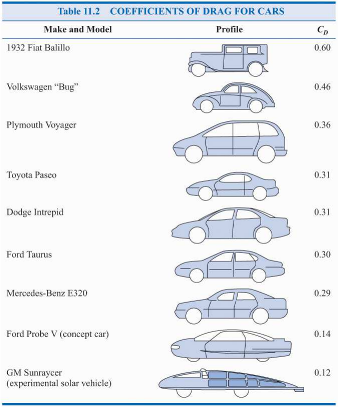


Table 11.2 COEFFICIENTS OF DRAG FOR CARS

Make and Model	Profile	C_D
1932 Fiat Balillo		0.60
Volkswagen "Bug"		0.46
Plymouth Voyager		0.36
Toyota Paseo		0.31
Dodge Intrepid		0.31
Ford Taurus		0.30
Mercedes-Benz E320		0.29
Ford Probe V (concept car)		0.14
GM Sunraycer (experimental solar vehicle)		0.12

Summary and Suggested Reading.

Lecture Summary

- Noted that boundary layer separation is caused by adverse pressure gradients.
- The pressure distribution around the object contributes to the momentum change like shear stress.
- Form (pressure) drag can have a major impact on the total drag budget of an object.
- Some objects, bluff bodies, have well defined separation points that are independent of Re.
- Practically, we almost always want to reduce drag, but are usually constrained by the object function.

Suggested Further Reading

- Crowe et al. '*Engineering Fluid Mechanics*', 9^{th} ed. SI version, Section 9.6, Chapter 11.
- Shrimpton, Section 5.7.

Suggested Further Study

- Start some chapter 12 questions. solution to 12.11 provided.